Signaling Schemes for Revenue Maximization

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(MSR and Technion)

Which **information** to **reveal** in the interface of AdExchange and how does that affect **revenue** and **welfare** ?

web surfers =

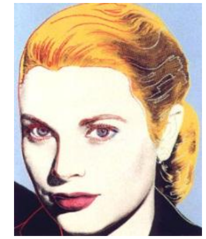
    

web surfers =

p_1

p_2

p_3



p_4

p_5

The New York Times

Tuesday, July 26, 2011 Last Update: 12:36 AM ET

Parties Head to Showdown as Obama Warns of a 'Crisis'

By CARL HULSE and JACKIE CALMES 39 minutes ago

President Obama urged "a balanced approach" that would include higher tax revenues as well as cuts. Speaker John A. Boehner accused Mr. Obama of asking for "a blank check."

- Video: President Obama's Address (MSNBC.com)

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- DealBook: Debt Drama Blocks Out Big Picture on Credit
- Gotham: Government Can't Help? Tell That to the South Bronx

Pool photo by Jim Watson, left; Chip Somodevilla/Getty Images

President Obama and Speaker John A. Boehner each spoke Monday.

NEWS ANALYSIS

A 'Unique Opportunity' on the Debt Ceiling, Lost

By JACKIE CALMES 9:53 PM ET

Whatever deal Congress and President Obama devise, it will probably fall short of the one nearly struck last week.

- FiveThirtyEight: Is Wall Street Out of Touch? 11:44 PM ET

OPINION »

THE DEBT STALEMATE

Editorial: Republican Wreckage

Their latest actions push the nation to the brink of default.

Brook: Congress Leads

As the Old Guard takes the lead, President Obama has faltered.

- Nocera: Wells Fargo's Pass
- Cohen: Breivik's Enablers
- Mike Mullen: Trust With China
- Op-Ed: Terror Within

MARKETS »

At 12:18 AM ET

JAPAN		CHINA
Nikkei	HangSeng	Shanghai
10,123.16	22,502.65	2,690.33
+73.15	+209.36	+1.58
+0.73%	+0.94%	+0.06%

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Microsoft Advertising Exchange

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holds a second price auction

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TIFFANY & CO.

b_1

Music Store

b_2

Pop Art Supplies

b_3

ad slot

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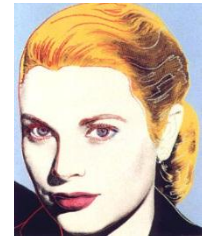
Their value depends who is the user behind the impression.

web surfers = {

				
p_1	p_2	p_3	p_4	p_5

}

TIFFANY & CO.	5	0.1	15	10	20

web surfers = {      }

	p_1	p_2	p_3	p_4	p_5
TIFFANY & CO.	5	0.1	15	10	20
 Pop Art Supplies	25	10	0.1	0.1	0.1

web surfers = $\{p_1, p_2, p_3, p_4, p_5\}$

	p_1	p_2	p_3	p_4	p_5
TIFFANY & CO.	5	0.1	15	10	20
Pop Art Supplies	25	10	0.1	0.1	0.1
Music Store	10	20	1	5	0.2

web surfers = $\left\{ \quad \right\}$

$p_1 \quad p_2 \quad p_3 \quad p_4 \quad p_5$

TIFFANY & CO.

$\vdots$

Pop Art
Supplies

$\cdots\cdots \quad v_i(j) \quad \cdots\cdots$

$\vdots$

Music
Store

Who knows what ?

- AdExchange knows who is the user **j** issuing the click

- Advertisers just know the prior **p**

One idea: revealing all the information

- Advertiser i bids $v_i(j)$
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- Advertiser i bids $v_i(j)$
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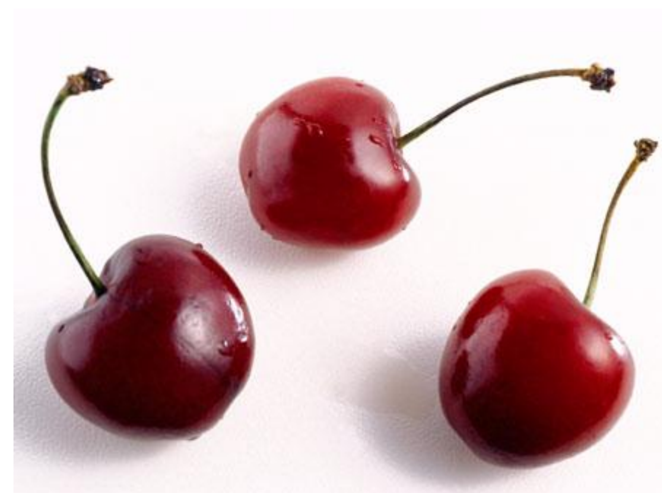
- Many problems:
  - Cherry picking
  - Revenue collapse
  - Adverse selection
  - Too much cognitive burden



web surfers = {  }

	p_1	p_2	p_3	p_4	p_5
TIFFANY & CO.	0.1	0.1	15	15	15
 Pop Art Supplies	25	0.1	0.1	0.1	0.1
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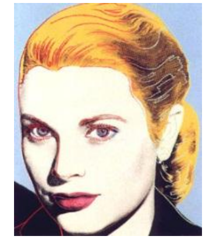
web surfers = { $p_1 + p_2$  p_3  p_4  p_5 }

	$p_1 + p_2$	p_3	p_4	p_5
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 Pop Art Supplies	13	0.1	0.1	0.1
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web surfers = $\{ \; p_1 + p_2 \; \; p_3 + p_4 + p_5 \; \}$

	$p_1 + p_2$	$p_3 + p_4 + p_5$
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 Pop Art Supplies	13	0.1
 Music Store	13	1

Other idea: bundling the items

- Group the items in sets $S_1 \ldots S_n$
- Revenue = $\sum_t \max2_i \sum_{j \in S_t} p(j) v_i(j)$

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Integral Partitioning Problem

Bundling the items fractionally

Bundling the items fractionally

=

Signaling

Bundling the items fractionally

$$=$$

Signaling

- [Emek, Feldman, Gamzu, Paes Leme, Tennenholtz '12]
- [Bro Miltersen, Sheffet '12]

Signaling

- Design a signal σ which is a random variable correlated with j

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- $\sigma \in [s]$ and is represented by a joint probability $\mathbb{P}(j,\sigma)$

$$\begin{array}{cc} & \begin{array}{ccc} \cdots & j & \cdots \end{array} \\ \begin{array}{c} \vdots \\ \sigma \\ \vdots \end{array} & \begin{bmatrix} & \vdots & \\ \cdots & \mathbb{P}(j,\sigma) & \cdots \\ & \vdots & \end{bmatrix} \end{array}$$

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$$
\begin{array}{cc}
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\begin{array}{c} \vdots \\ \sigma \\ \vdots \end{array} & \begin{bmatrix} & \vdots & \\ \cdots & \mathbb{P}(j, \sigma) & \cdots \\ & \vdots & \end{bmatrix}
\end{array}
\qquad
\boxed{\sum_\sigma \mathbb{P}(j, \sigma) = p(j)}
$$

Signaling

- For user **j**, the search engine samples σ according to

$$\mathbb{P}(\sigma|j) = \frac{\mathbb{P}(j,\sigma)}{\sum_{j'} \mathbb{P}(j',\sigma)}$$

- Advertiser use σ to update their bid

$$b_i = \mathbb{E}[v_i(j)|\sigma] = \frac{\sum_j v_i(j)\mathbb{P}(j,\sigma)}{\sum_j \mathbb{P}(j,\sigma)}$$

p_1

p_2

p_3

p_4

p_5

j=3

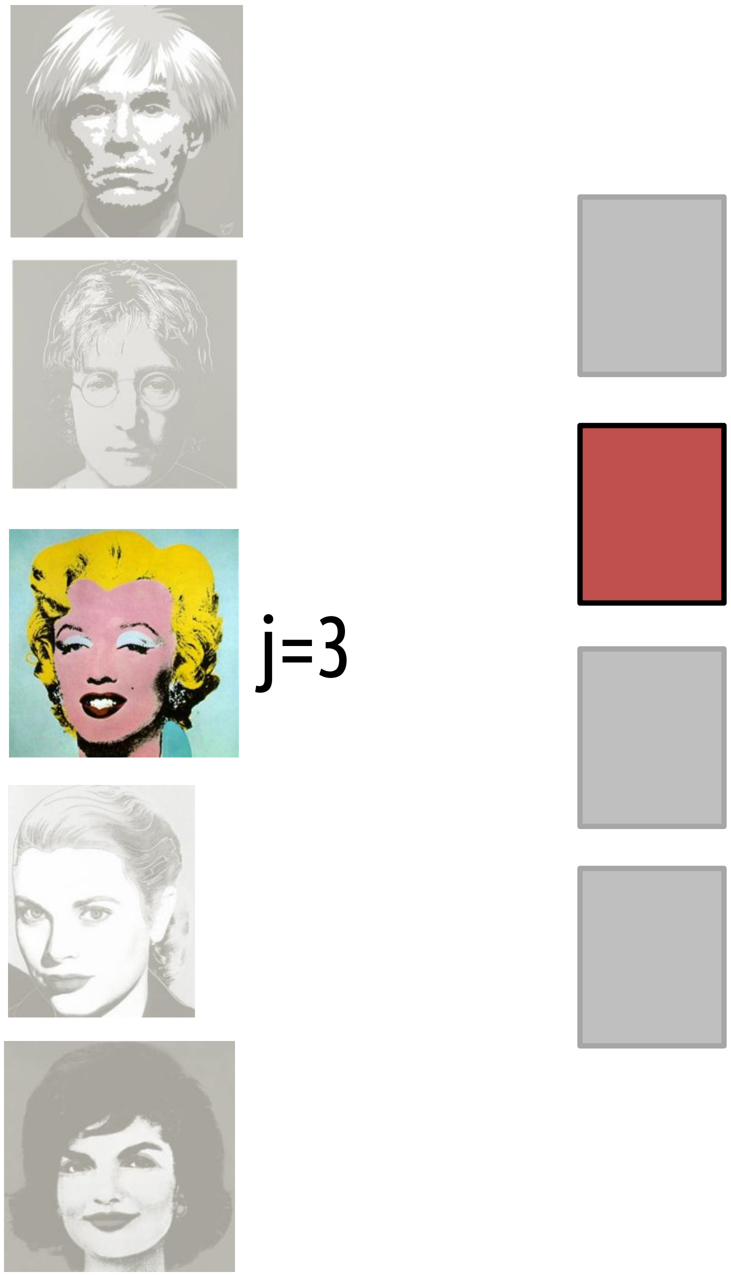
j=3

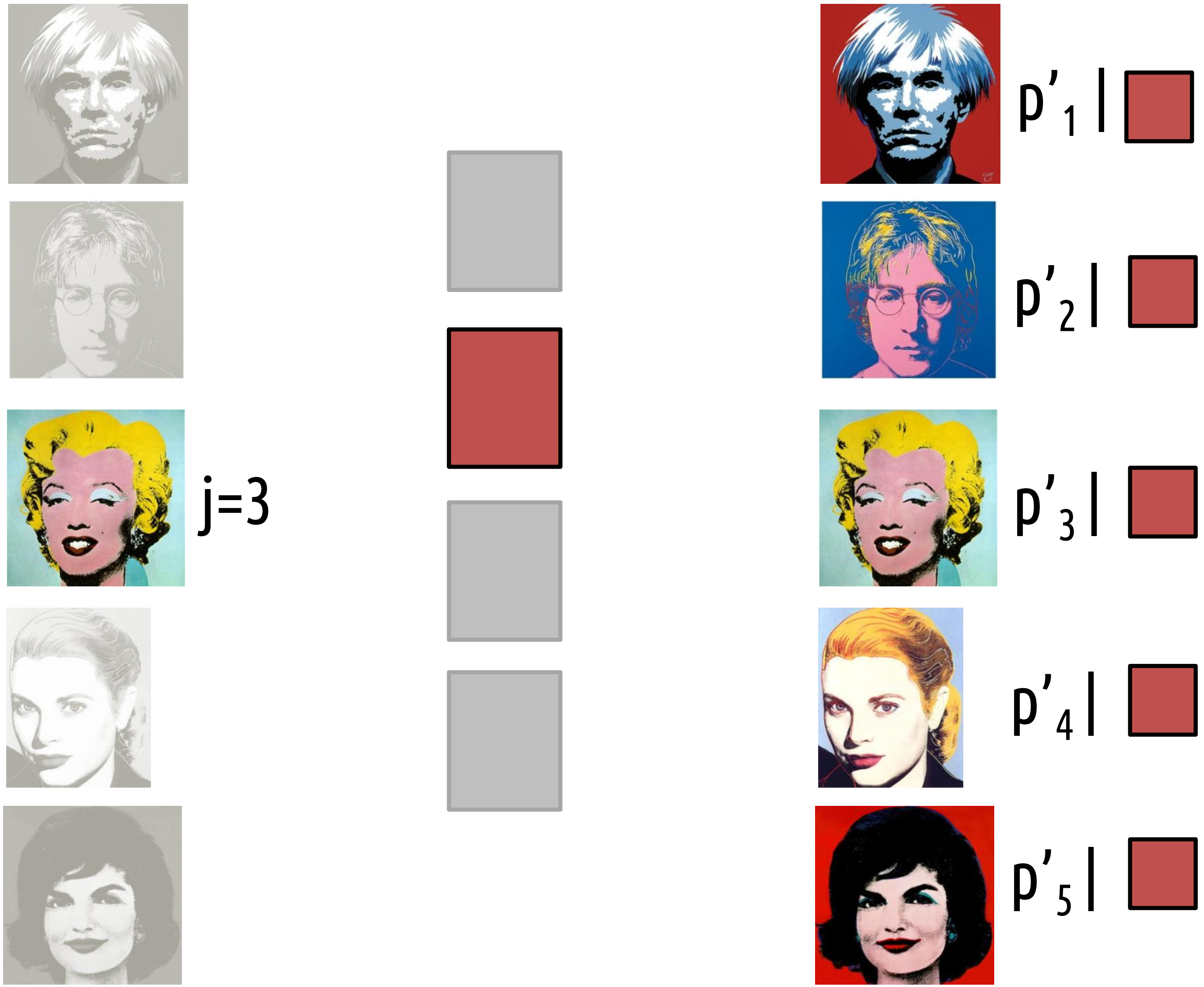
$$\mathbb{P}(\sigma_1 | j = 3)$$

$$\mathbb{P}(\sigma_2 | j = 3)$$

$$\mathbb{P}(\sigma_3 | j = 3)$$

$$\mathbb{P}(\sigma_4 | j = 3)$$


j=3


j=3
p'1 |
p'2 |
p'3 |
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p'5 |

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  $= \sum_\sigma \mathrm{max2}_i \sum_j v_i(j)\mathbb{P}(j,\sigma)$

- How big does **s** (size of signaling space) need to be ?

- How to optimize revenue ? (max2 is not convex)

Signaling

- Theorem: If there are n advertisers, we just need to keep **n (n-1)** signals. One correspond to each pair of advertisers (i_1, i_2)

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$$\max \sum_{i_1,i_2\in[n],i_1\neq i_2} R(\sigma_{i_1,i_2}) \text{ s.t.}$$

$$R(\sigma_{i_1,i_2}) \leq \sum_{j\in[m]} \mathbb{P}(j,\sigma_{i_1,i_2})\cdot v_{i_1}(j) \quad \forall i_1,i_2\in[n], i_1\neq i_2$$

$$R(\sigma_{i_1,i_2}) = \sum_{j\in[m]} \mathbb{P}(j,\sigma_{i_1,i_2})\cdot v_{i_2}(j) \quad \forall i_1,i_2\in[n], i_1\neq i_2$$

$$\sum_{i_1,i_2\in[n],i_1\neq i_2} \mathbb{P}(j,\sigma_{i_1,i_2}) = p(j) \ \forall j\in[m]$$

$$\mathbb{P}(j,\sigma_{i_1,i_2}) \geq 0 \quad \forall i_1,i_2\in[n], i_1\neq i_2, \forall j\in[m] \ .$$

Signaling

- Theorem: The revenue-optimal signaling can be found in polynomial time.

- Also, there is an optimal signaling scheme that preserves ½ of the optimal social welfare.

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- Theorem: The revenue-optimal signaling can be found in polynomial time.
- Also, there is an optimal signaling scheme that preserves ½ of the optimal social welfare.
- It improves the optimal (integral) bundling up to a factor of 2.

Signaling in a Bayesian World

- Valuations of advertiser i for user j depends on some unknown state of the world $\omega \sim q$

$$v_i(j, \omega)$$

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- Valuations of advertiser i for user j depends on some unknown state of the world $\omega \sim q$

$$v_i(j, \omega)$$

- Let $k = \mathrm{supp}(q)$
- We can find the optimal signaling scheme in polynomial time if $k = O(1)$
  - Naïve extension of the full information LP

Signaling in a Bayesian World

- If m (number of user types) is constant, then we can find the optimal signaling scheme in time polynomial in k,n.
  - Geometry of hyperplane arrangements

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- Open: approximability of this problem

Open Problems

Approximability in the Bayesian Case

Open Problems

Approximability in the Bayesian Case

Bayesian case with independent values

Open Problems

Approximability in the Bayesian Case

Bayesian case with independent values

Optimal auctions with signaling

Thanks !