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Renato Paes Leme Éva Tardos

Cornell University

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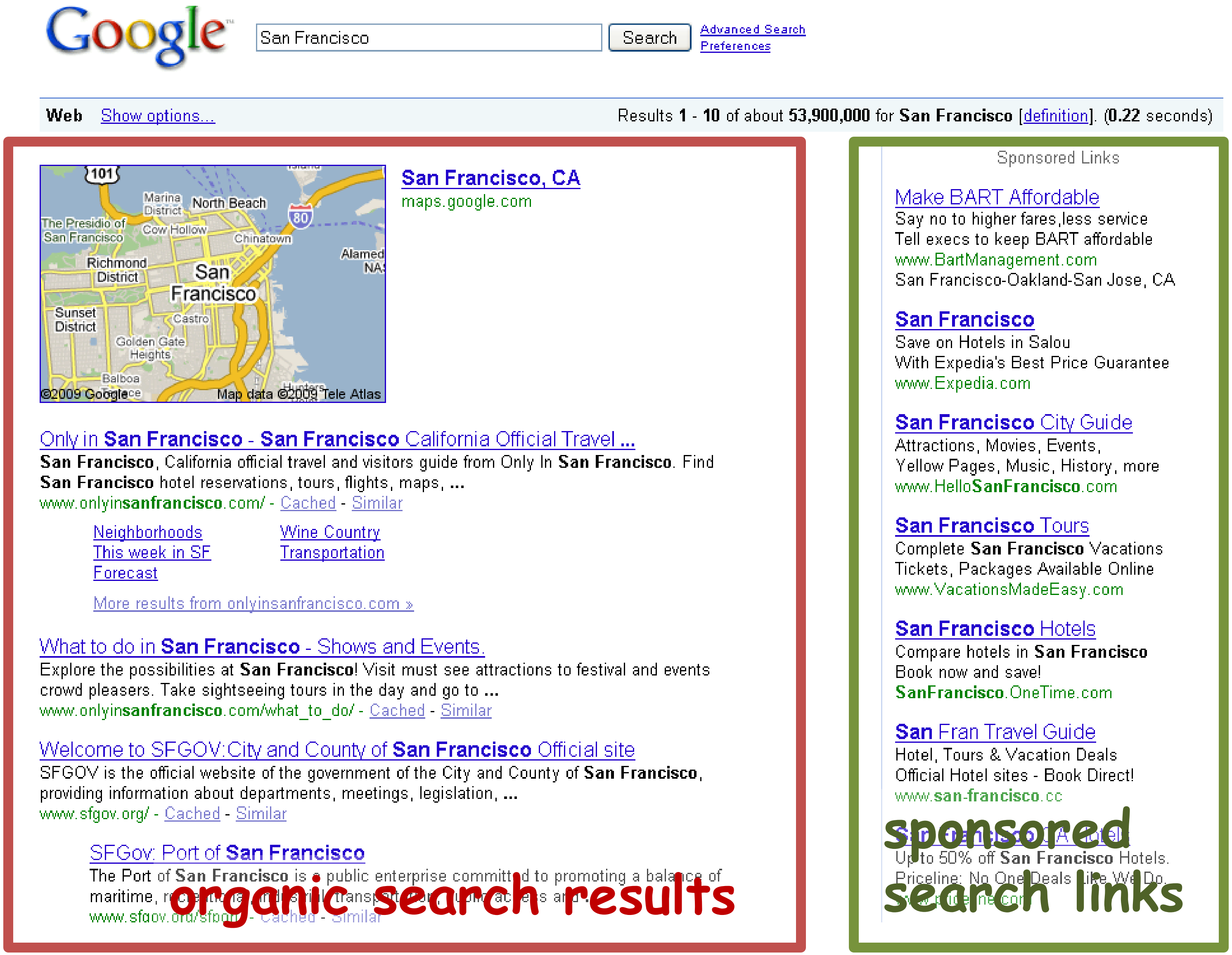

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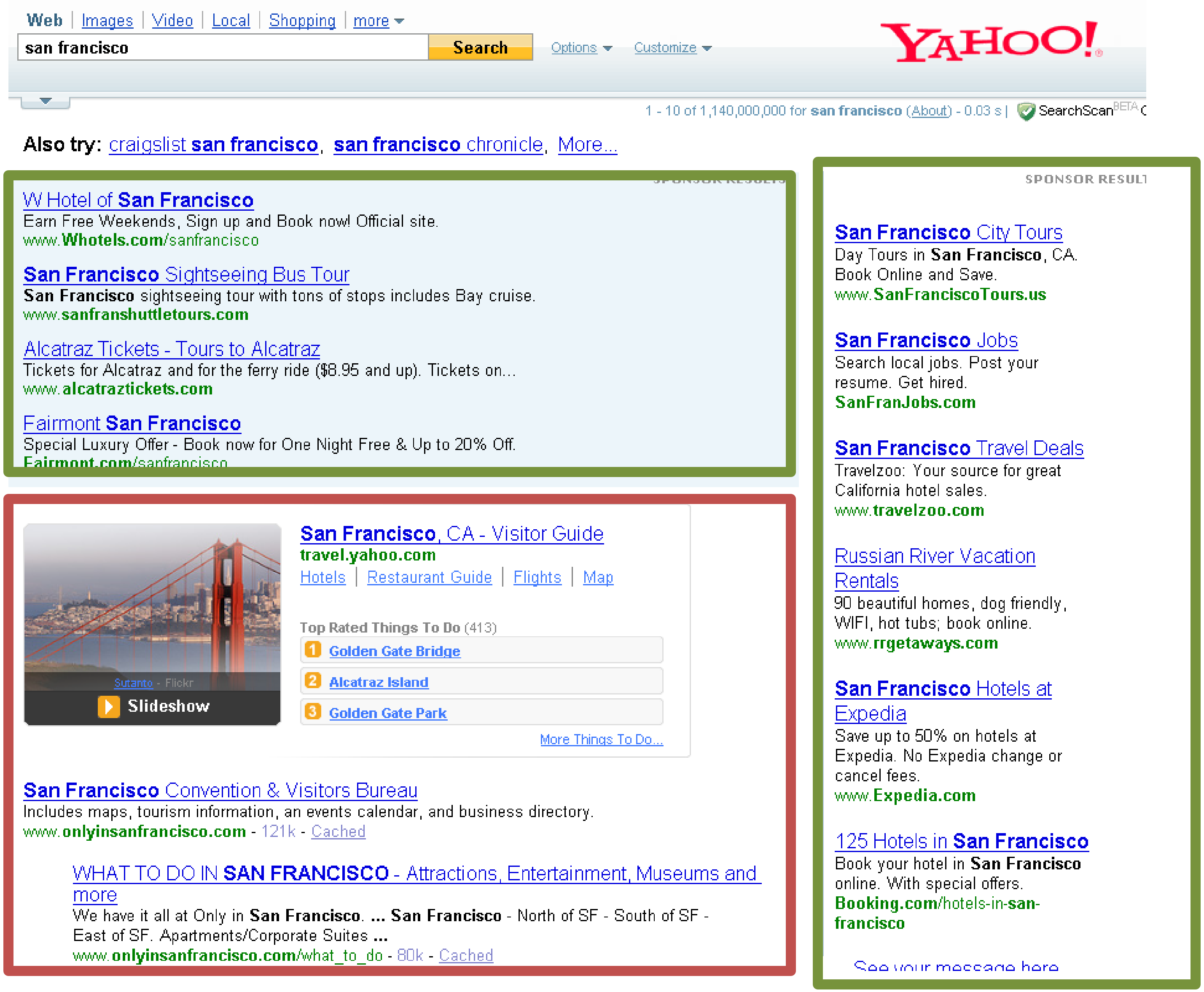

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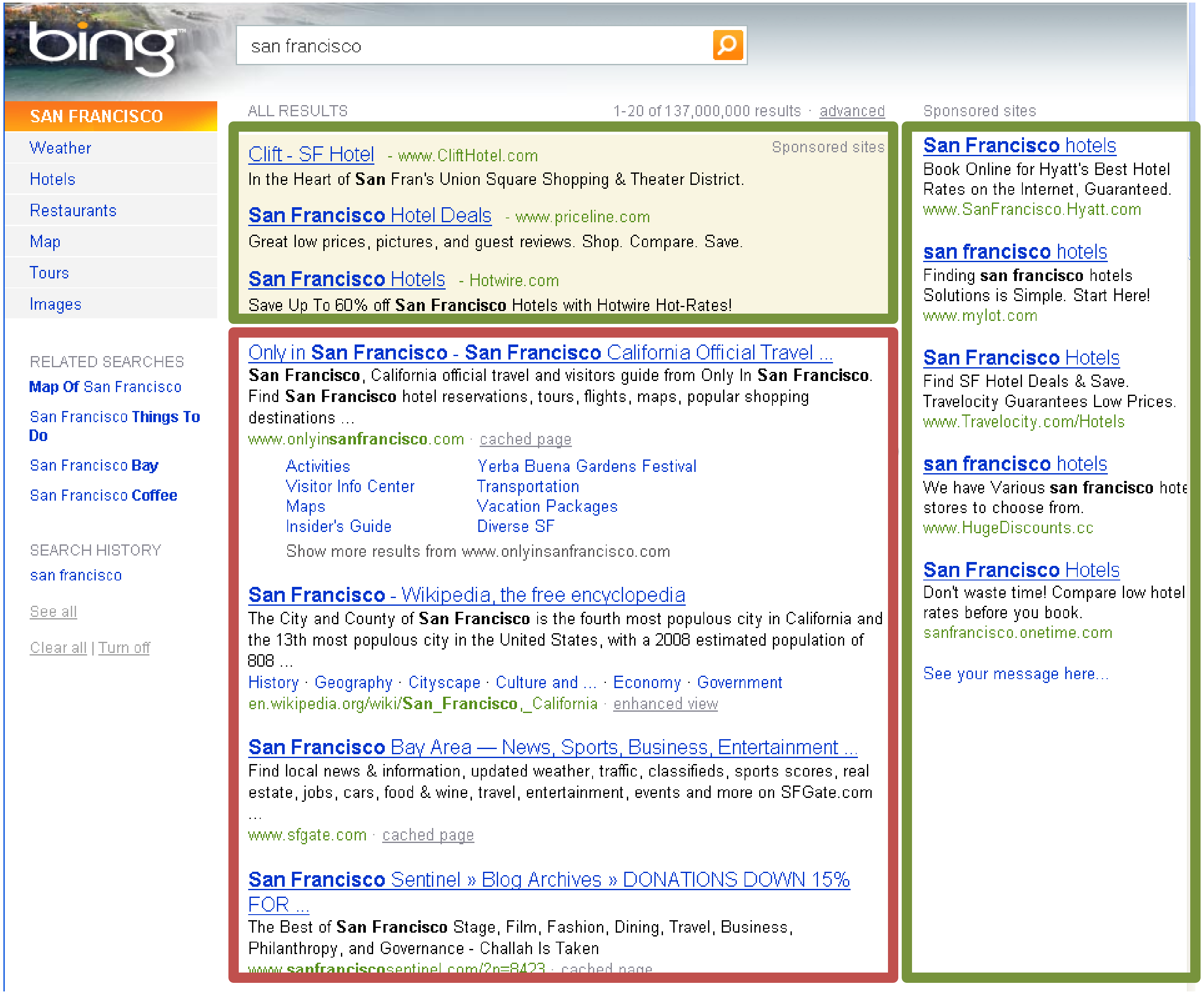

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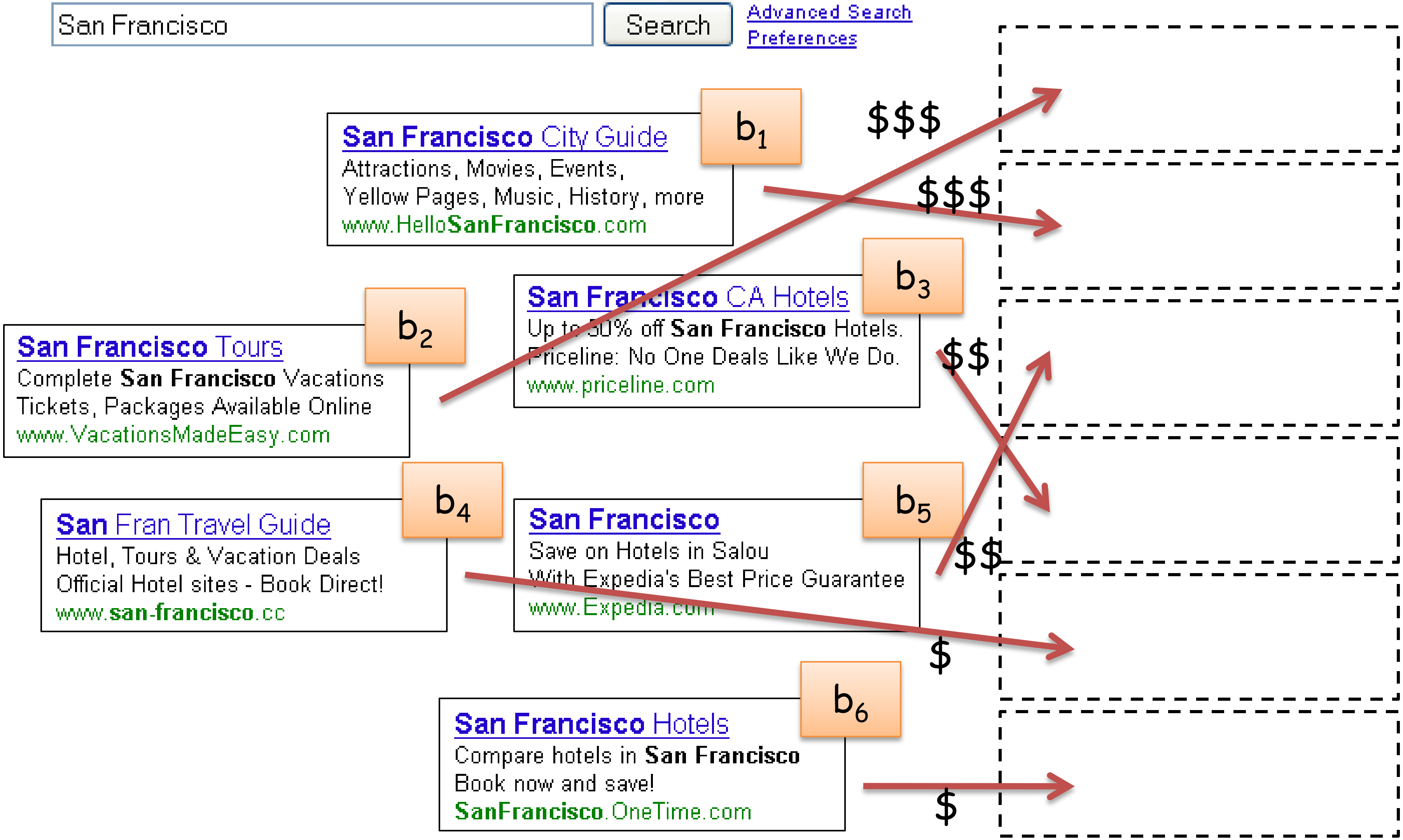
Auction Model



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Auction Mechanisms

~~VCG~~

~~Myerson's Mechanism~~

~~Reserve prices~~

GSP

- Not truthful
- Doesn't maximize Social Welfare
- Simple and Natural
- Good balance between revenue and social welfare

Main Result

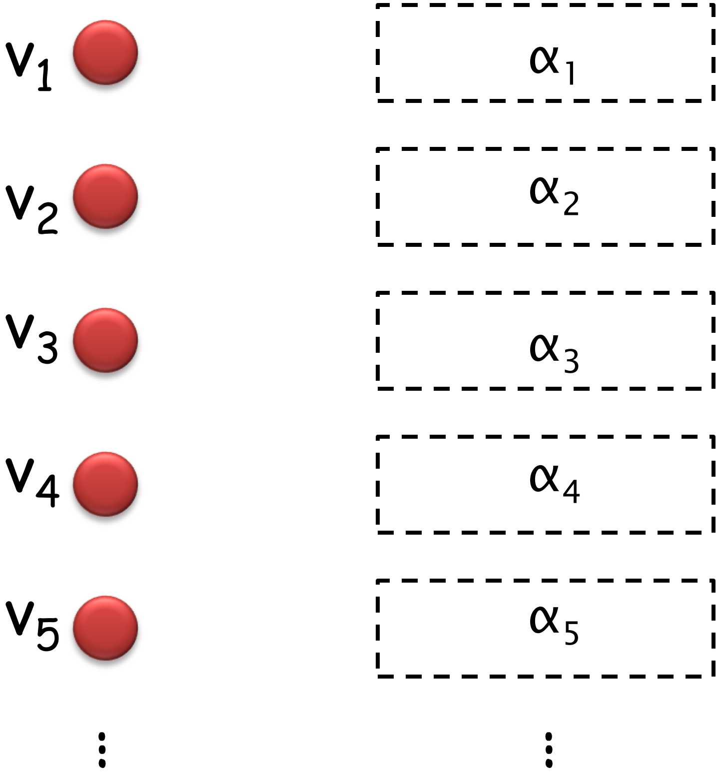
Under some assumptions, any Nash equilibrium in GSP is within a factor of 1.618 to the optimal social welfare.

Golden ratio: $\frac{1+\sqrt{5}}{2} \approx 1.618$

Today we prove a factor of 2.

Model

- n advertisers and n slots
- Each advertiser has a value v_i
- Each advertiser submits a bid b_i
- Each slot has a click-through-rate α_i

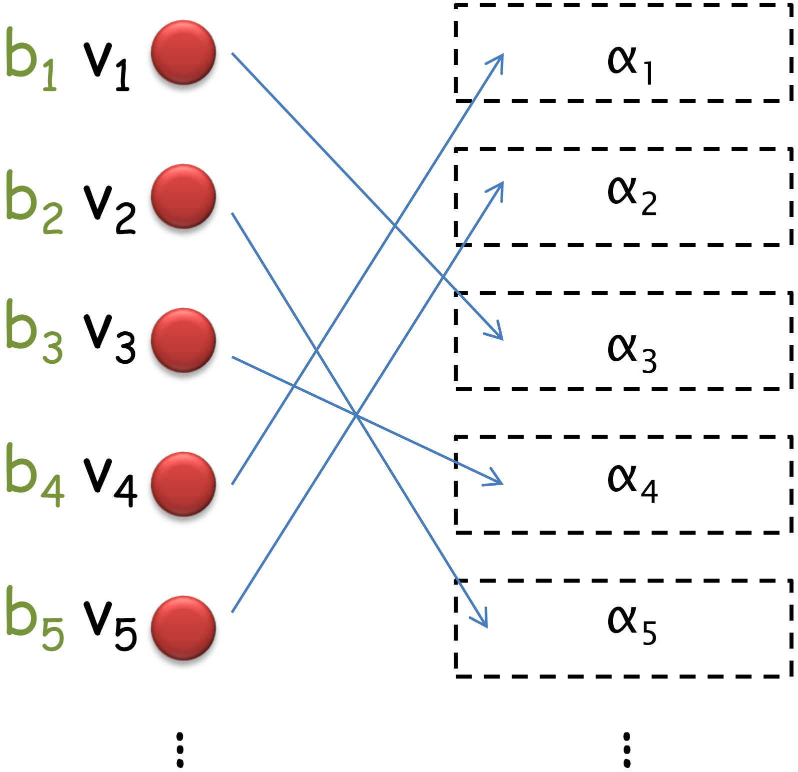


$$v_1 \geq v_2 \geq \ldots \geq v_n$$

$$\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_n$$

Model

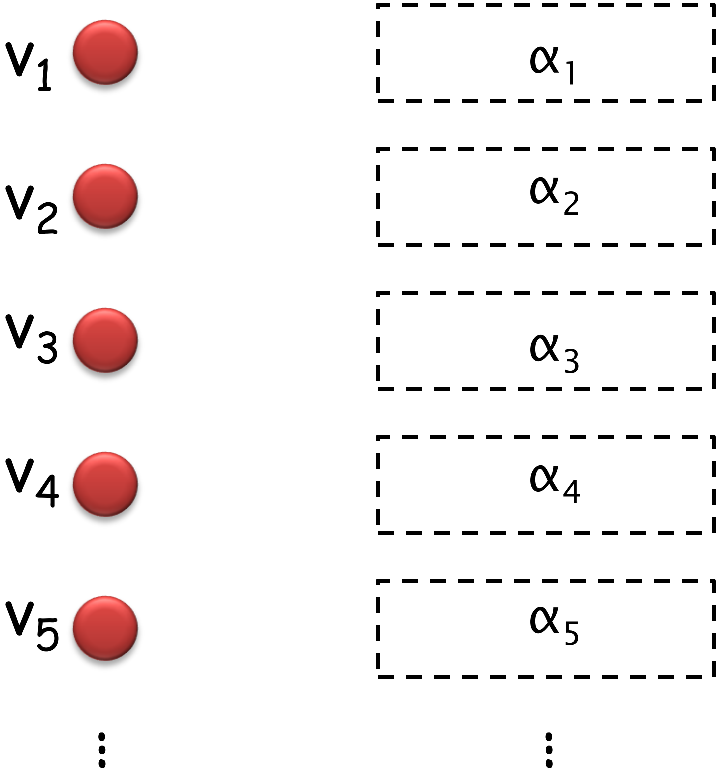
- Advertisers are ordered by bids and assigned to slots
- They are charged the next highest bid



$$v_1 \geq v_2 \geq \dots \geq v_n$$
$$\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_n$$

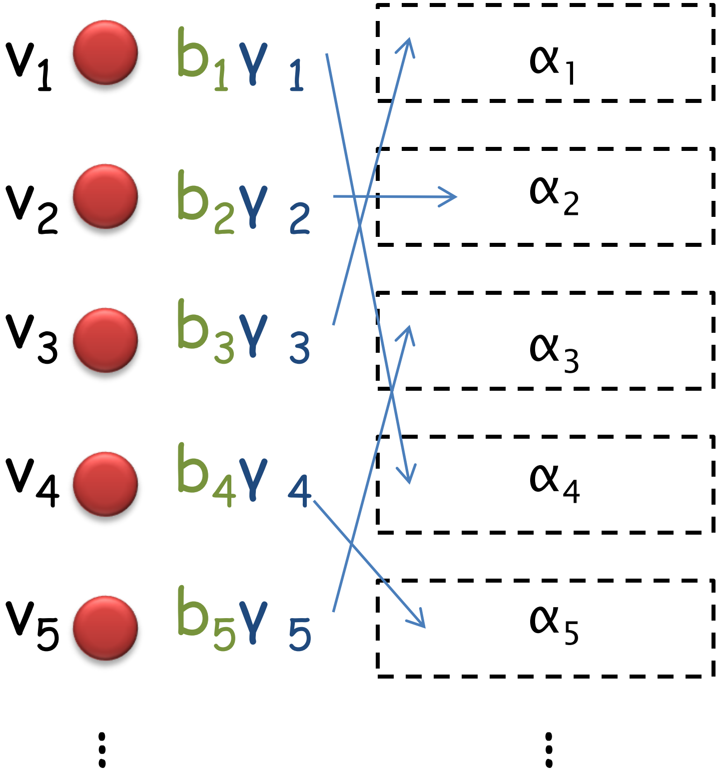
Model

- Utility of player i when assigned to slot j:
  $u_i = \alpha_j(v_i - b_{\pi(j+1)})$
- Allocation π
  $\pi(j)$ is the bidder allocated in slot j



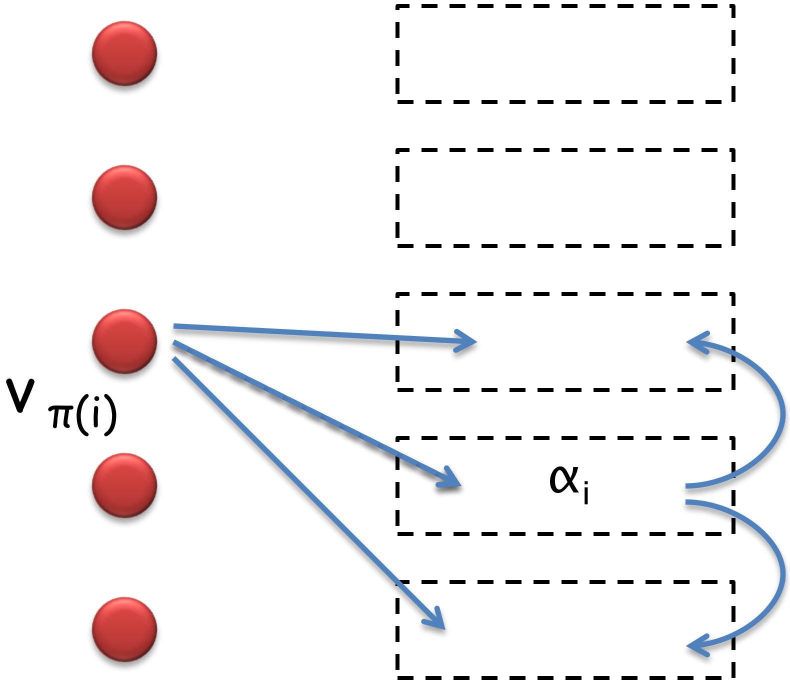
Separable Click Through Rates

- More general model
- Quality score γ
- Same bounds
- Today: stick with simplest model



Nash Equilibrium

- A set of bids $(b_1, \ldots, b_n)$ and its corresponding assignment π is a Nash equilibrium if:



$$\alpha_i (v_{\pi(i)} - b_{\pi(i+1)}) \geq \alpha_j (v_{\pi(i)} - b_{\pi(j)}) \qquad j < i$$
$$\alpha_i (v_{\pi(i)} - b_{\pi(i+1)}) \geq \alpha_j (v_{\pi(i)} - b_{\pi(j+1)}) \qquad j > i$$

Nash Equilibrium

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$$\alpha_i (v_{\pi(i)} - b_{\pi(i+1)}) \geq \alpha_j (v_{\pi(i)} - b_{\pi(j+1)}) \quad j > i$$

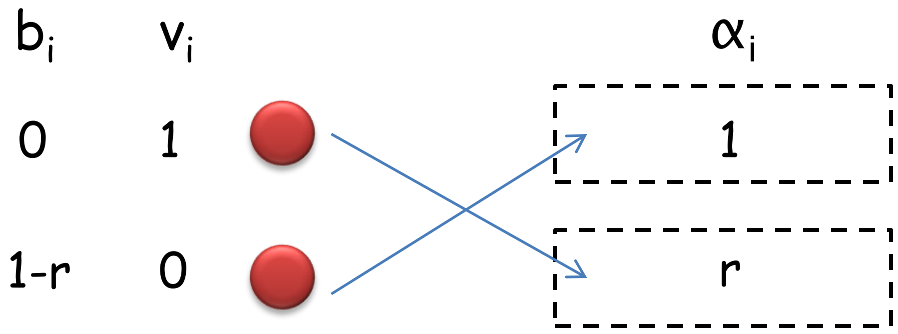
- Social Welfare of an assignment:

$$SW = \sum_j \alpha_j v_{\pi(j)}$$

There are good equilibria…

- Theorem [Edelman & Ostrovsky & Schwarz, Varian]: There is always a Nash equilibrium for GSP maximizing social welfare.

… and bad equilibria



This is a Nash equilibrium with Social Welfare = r.
Optimum Social Welfare = 1.

⇨ Arbitrarily large gap: $1/r \rightarrow \infty$

But this configuration is very unnatural, since the second player is taking a lot of risk.

Conservative Assumption

- Assuming bidders are conservative, i.e., no one bids above its valuation:

$$b_i \leq v_i$$

we can prove that each Nash is within a factor of 1.618 to the optimal.

⇨ Price of anarchy: $\frac{SW\ (OPT)}{SW(Nash)}$

Conservative Assumption

- Assuming bidders are conservative, i.e., no one bids above its valuation:

$$b_i \leq v_i$$

we can prove that each Nash is within a factor of 1.618 to the optimal.

- Related result: [Lahaie] proves a bound on the price of anarchy supposing a good separation of the click-through-rates.

Weakly feasible assignment

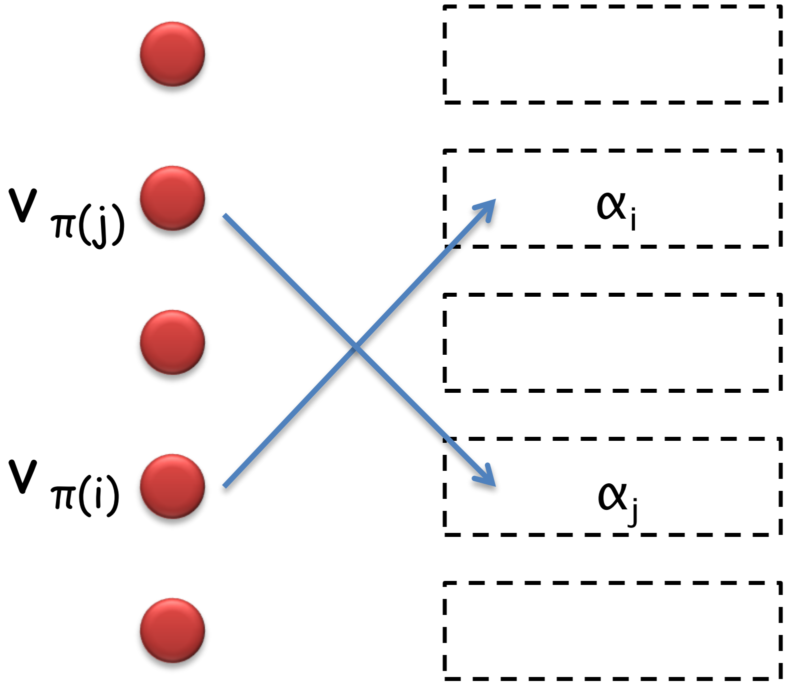
Lemma: If π is an allocation in a Nash equilibrium under the conservative assumption, then:

$$\frac{\alpha_j}{\alpha_i} + \frac{v_{\pi(i)}}{v_{\pi(j)}} \geq 1$$

therefore:

Weakly feasible assignments

$$\frac{\alpha_j}{\alpha_i} \geq \frac{1}{2} \quad \text{or} \quad \frac{v_{\pi(i)}}{v_{\pi(j)}} \geq \frac{1}{2}$$



Weakly feasible assignment

Lemma: If π is an allocation in a Nash equilibrium under the conservative assumption, then:

$$\frac{\alpha_j}{\alpha_i} + \frac{v_{\pi(i)}}{v_{\pi(j)}} \geq 1$$

Proof: Need to prove only if $i < j$ and $\pi(i) > \pi(j)$. It is a combination of 3 relations:

$\alpha_j (v_{\pi(j)} - b_{\pi(j+1)}) \geq \alpha_i (v_{\pi(j)} - b_{\pi(i)})$ [Nash]

$b_{\pi(j+1)} \geq 0$ $\qquad$ $b_{\pi(i)} \leq v_{\pi(i)}$ [conservative]

□

Some intuition…

$$\frac{\alpha_j}{\alpha_i} + \frac{v_{\pi(i)}}{v_{\pi(j)}} \geq 1$$

- If values v_i are very close then their order doesn't influence social welfare much
- If values v_i are well separated, then permutations producing bad social welfare are not weakly feasible

More symmetric and easy to use.

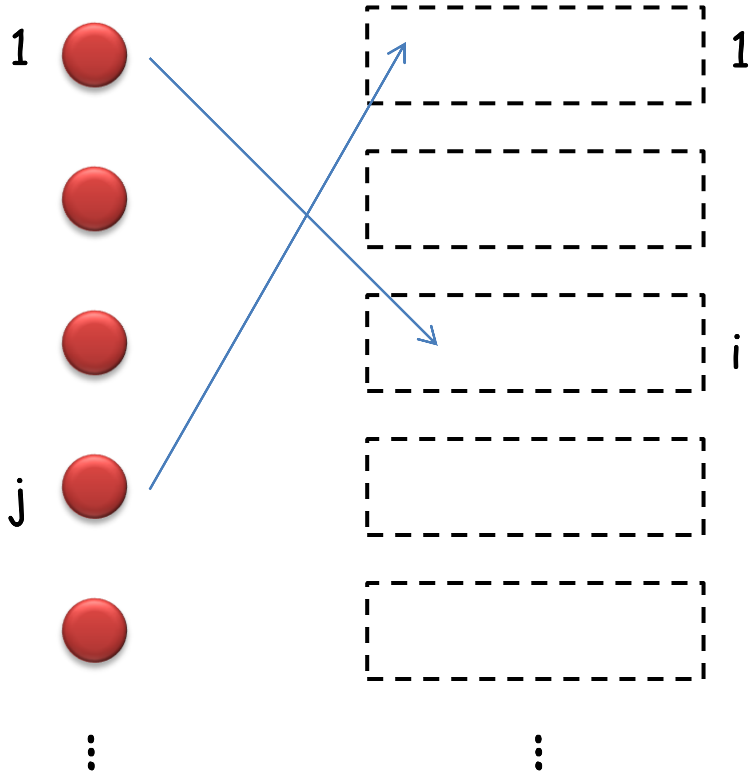
Factor of 2

Theorem: Any conservative Nash equilibrium is within a factor of 2 to the optimum.

Theorem: Any weakly feasible assignment is within a factor of 2 to the optimum.

Factor of 2

Proof: Induction on the number of slots.



By the lemma:

$$\frac{\alpha_i}{\alpha_1} \geq \frac{1}{2} \text{ or } \frac{v_j}{v_1} \geq \frac{1}{2}$$

In the first case, remove bidder 1 and slot i and apply inductive hypothesis

Factor of 2

Proof: Applying the induction hypothesis:.

$$\begin{aligned}\Sigma_{k\neq i}\alpha_k v_{\pi(k)} &\geq \tfrac{1}{2}\,(\alpha_2 v_1 + \ldots + \alpha_i v_{i-1} + \alpha_{i+1} v_{i+1} + \ldots + \alpha_n v_n) \\ &\geq \tfrac{1}{2}\,(\alpha_2 v_2 + \ldots + \alpha_i v_i + \alpha_{i+1} v_{i+1} + \ldots + \alpha_n v_n)\end{aligned}$$

$$\Sigma_k \alpha_k v_{\pi(k)} = \alpha_i v_1 + \Sigma_{k\neq j}\alpha_k v_{\pi(k)} \geq \tfrac{1}{2}\,\alpha_1 v_1 + \tfrac{1}{2}\,\Sigma_{k>1}\alpha_k v_k$$

$\square$

Using the Lemma in its full potential
gives us the 1.618 bound.

What else can we do:

- Bound of 1.618
- Same bounds for separable click-through-rates: quality score
- Similar bounds for γ-conservative bidders: $\gamma b_i \leq v_i$