Non Clairvoyant Dynamic Mechanism Design

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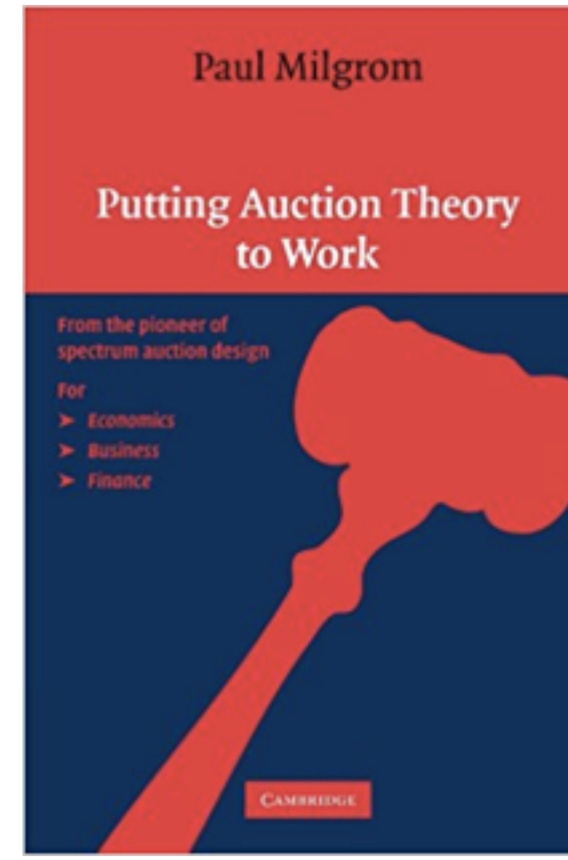
Next generation of ad auction

- Classic auctions found their way to the web
- Designed for different domains: art, spectrum, …



- Internet ad auctions are different: repeated and the buyer cares about the aggregate result.

- Why use dynamic auctions ?
  - Can improve both revenue and efficiency over static auctions (no tradeoffs)
  - Can generate arbitrarily more revenue than static auctions.
  - Combines the best of real time auctions and guaranteed contracts.

Towards practical dynamic auctions

- Current state:
  - beautiful mathematical theory [...]
  - polynomial time algorithms [PPPR], [ADH]
  - understanding of competition complexity [LP]

- Barriers to a practical implementation:
  - DP / LP solutions are not scalable
  - relies on accurate forecasts
  - assumes too much of buyer rationality / knowledge

Repeated Auctions Model

- Single buyer model

- For timestep t = 1…T
  - item arrives (ad impression)
  - buyer observes his type $v_t \sim F_t$
    (sellers <- public info, buyer <- public info + private cookies)
  - agent reports value $\hat{v}_t$
  - allocation with probability $x_t(\hat{v}_{1..t})$ and pays $p_t(\hat{v}_{1..t})$
  - buyer gets utility $u_t^{v_t} = v_t x_t(\hat{v}_{1..t}) - p_t(\hat{v}_{1..t})$

- Buyer wants to maximize continuation utilities

$$u_t^{v_t}(\hat{v}_{1..t}; F_{1..T}) + \mathbb{E}_{F_{t+1..T}}\left[\sum_{\tau=t+1}^{T} u_\tau^{v_\tau}(\hat{v}_{1..\tau}; F_{1..T})\right]$$

Design Space

- The auction is represented by allocation and payments:

$$x_t : \Theta^t \times (\Delta\Theta)^T \to [0,1] \qquad x_t(\hat{v}_{1..t}; F_{1..T})$$

$$p_t : \Theta^t \times (\Delta\Theta)^T \to \mathbb{R}_+ \qquad p_t(\hat{v}_{1..t}; F_{1..T})$$

- Constraints:
  - Dynamic Incentive Compatibility (DIC)

    $$v_t \in \mathrm{argmax}_{\hat{v}_t} u_t^{v_t}(\hat{v}_t \ldots) + \mathbb{E}_{F_{t+1..T}}[\textstyle\sum_{\tau=t+1}^{T} u_\tau^{v_\tau}(\hat{v}_t \ldots)]$$

  - Ex-post Individual Rationality (ep-IR) $\sum_t u_t \geq 0$

- Objective function: $\textsc{Rev}^*(F_{1..T}) = \max \mathbb{E}_{F_{1..T}}[\sum_t p_t(v_{1..t})]$

Cassandra's curse

- Optimal mechanism requires seller to know all distributions in advance (to solve the DP).
- The definition of DIC require buyer and seller to agree on distributions $F_{t+1}, F_{t+2}, \ldots, F_T$.
- Can we get mechanism that doesn't require common knowledge about the future ?

- DIC:

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- Super-DIC: for any beliefs $\tilde{F}_{t+1..T}$
  $v_t \in \mathrm{argmax}_{\hat{v}_t} u_t^{v_t}(\hat{v}_t \ldots) + \mathbb{E}_{\tilde{F}_{t+1..T}}[\sum_{\tau=t+1}^{T} u_\tau^{v_\tau}(\hat{v}_t \ldots)]$
- Theorem (Cassandra's curse): Under super-DIC the revenue optimal mechanism is the optimal static auction.

Non-Clairvoyance

- **Non-Clairvoyance**: mechanism is measurable with respect to i.e. $v_{1..t}, F_{1..t}$ $\quad x_t(v_{1..t}; F_{1..t}), p_t(v_{1..t}; F_{1..t})$

- Entangled design: consider two items sequences:

  $$[\,F_a \quad F_g\,] \qquad [\,F_a \quad F_o\,]$$

  the non-clairvoyant mechanism needs to use the same rule for item 1. The clairvoyant can use different rules depending on what comes next.

- **DIC for Non-Clairvoyant**: buyers don't need to know the future to check DIC. The only requirement is that distribution F_t will be common knowledge in step t.

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Non Clairvoyant Revenue Approx

- **Benchmark**: the optimal dynamic mechanism that knows all the distributions $\textsc{Rev}^*(F_{1..T})$.

- A NonClairvoyant auction is an α-approximation if for all distributions $F_{1..T}$ we have
$$\textsc{Rev}(F_{1..T}) \geq \alpha \textsc{Rev}^*(F_{1..T})$$

Non Clairvoyant Revenue Approx

Theorem: Every non-clairvoyant policy is at most a 1/2-approximation to the optimal clairvoyant revenue.

Theorem: For multiple buyers there is a non-clairvoyant policy that is at least 1/5-approx to the opt clairvoyant.

Theorem: Can be improved to 1/2 for two periods and for 1/3 for one buyer and multiple periods.

Technique: Bank Account Mechanisms

Theorem: Every non-clairvoyant policy is "isomorphic" to a bank account mechanism.

- Keeps a state variable b_t (balance) for each buyer
- Chooses a per-period IC mechanism based on balance

$$x_t(v_t, b_t), p_t(v_t, b_t)$$

with the balance-independence property

$$\mathbb{E}[v_t x_t(v_t, b_t) - p_t(v_t, b_t)] = \text{const} \geq 0$$

- Updates balance:

$$0 \leq b_{t+1} \leq b_t + [v_t x_t - p_t]$$

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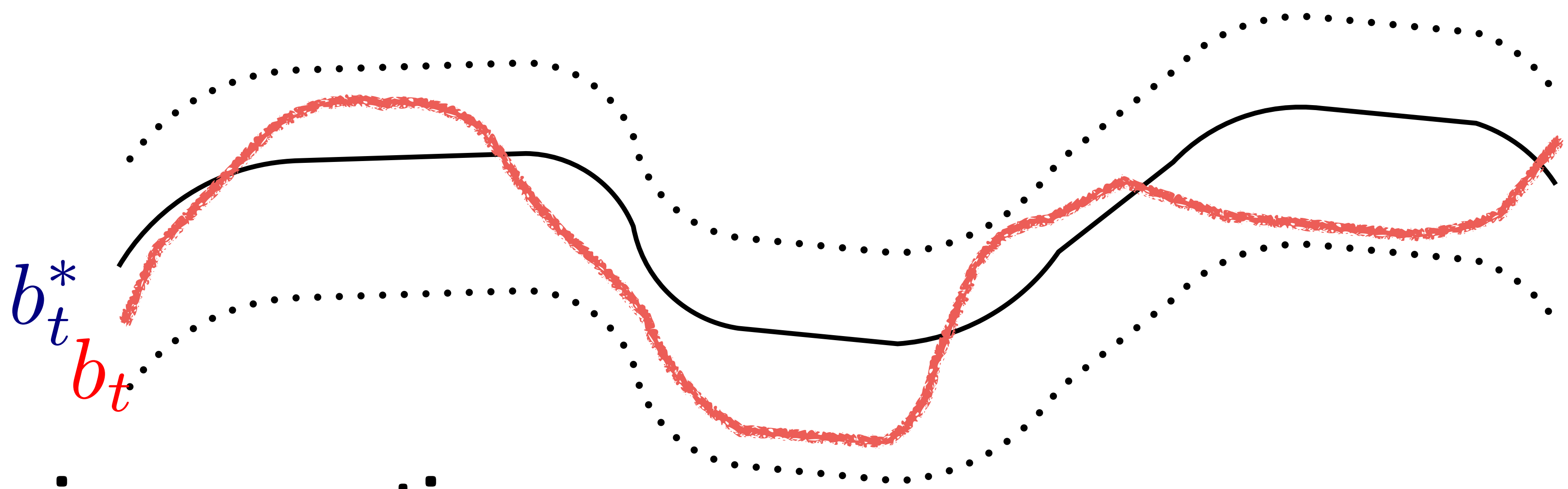

Other nice properties:

- framework to design and prove lower bounds on dynamic mechanisms
- computationally efficient (multi-buyer, multi-item)
- no pre-processing required (LP or DP)

1/3-approximation policy

Keep a variable b called balance initialized to zero.

For every period t, receive an item with distribution F_t

Sell 1/3 of the item with each of the following auctions:

- Myerson's auction for F_t
- Give the item for free and increment balance $b = b + v_t$
- For $f = \min(b, \mathbb{E}_{F_t}[v_t])$
  - charge f before the buyer can see the item
  - post a price of r such that $\mathbb{E}(v_t - r)^+ = f$
  - decrement balance $b = b - f$

Balance independence property: E[utility] is balance independent.

Extension to Multiple buyers

Single buyers (1/3 approx)	**Multiple buyers** (1/5 approx)
1/3 item: Myerson	1/3 item: Myerson
1/3 item: Give for free	2/3 item: Second price auction
1/3 item: Dynamic posted price	2/3 item: Dynamic money burning auction [HR]

Thanks

Non Clairvoyant Mechanism Design

https://ssrn.com/abstract=2873701