POLYHEDRAL CLINCHING AUCTIONS BEYOND HARD BUDGET CONSTRAINTS

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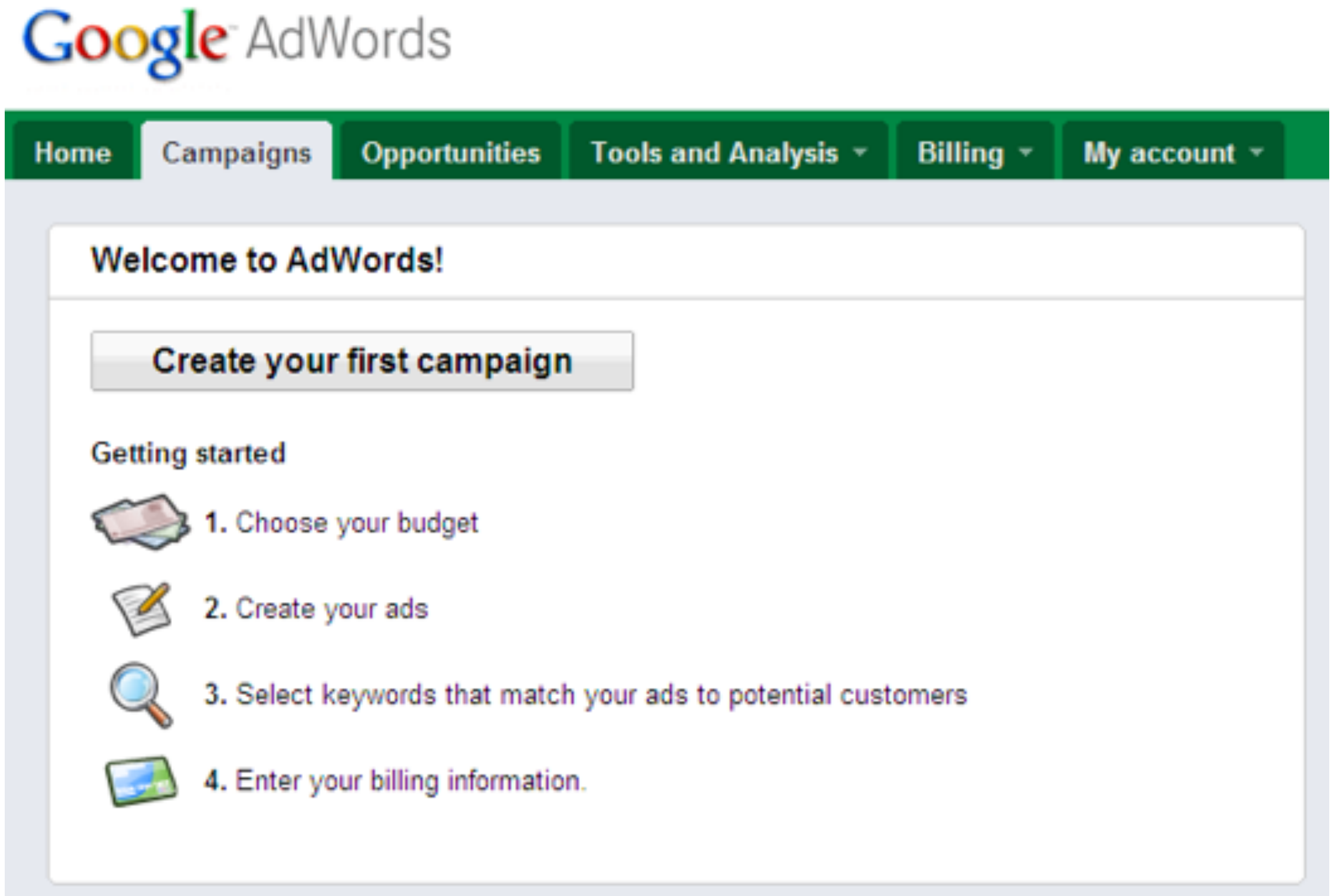
- **Item values** are an useful abstraction but often intangible.
- Typically, buyers care about the items (impressions) only in **aggregate**.
- Aggregate statistics about an auction result: **budget spent**, **average cpc**, ...


Google AdWords
Home
Campaigns
Opportunities
Tools and Analysis
Billing
My account
Welcome to AdWords!
Create your first campaign
Getting started
1. Choose your budget
2. Create your ads
3. Select keywords that match your ads to potential customers
4. Enter your billing information.

Bidding and budget

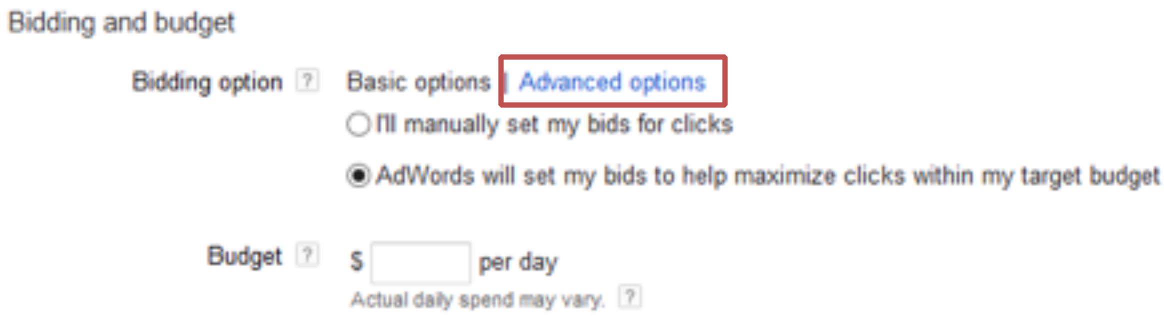
Bidding option [?] Basic options | Advanced options

( ) I'll manually set my bids for clicks

(•) AdWords will set my bids to help maximize clicks within my target budget

Budget [?] $ [   ] per day

Actual daily spend may vary. [?]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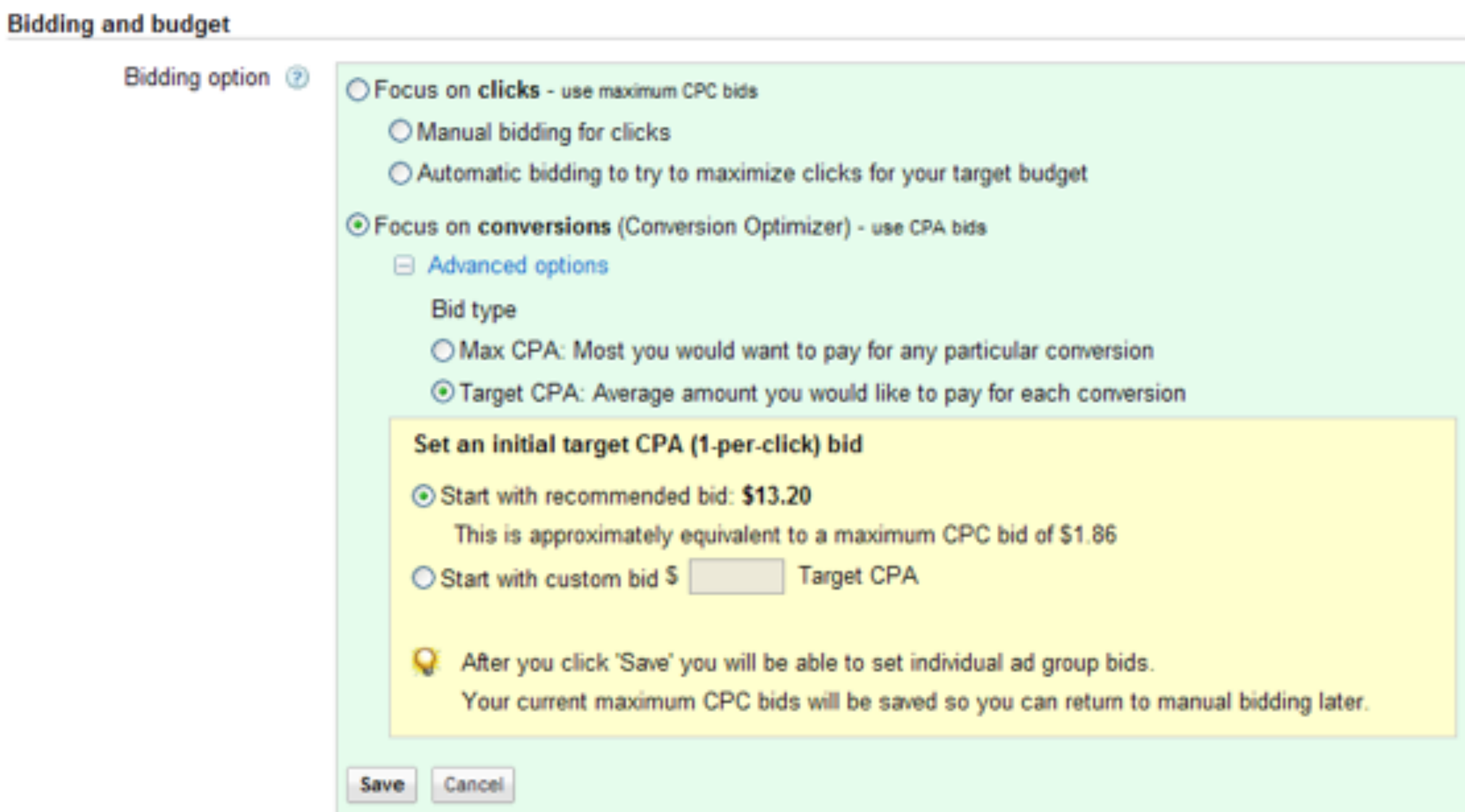
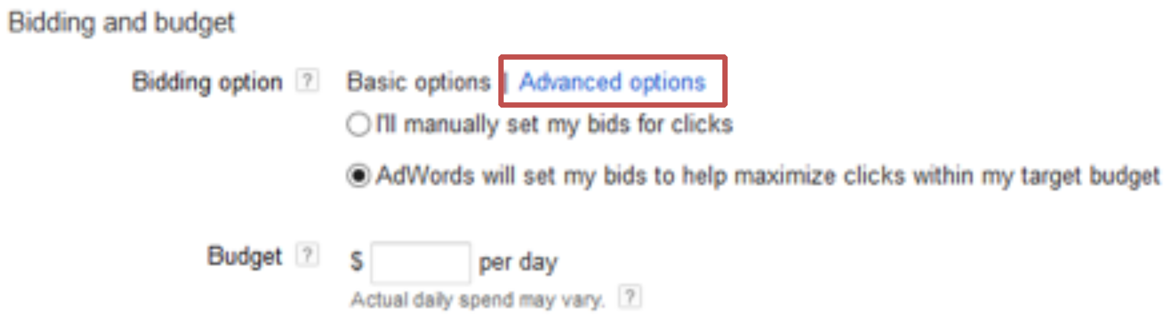

Bidding and budget
Bidding option
Focus on clicks - use maximum CPC bids
Manual bidding for clicks
Automatic bidding to try to maximize clicks for your target budget
Focus on conversions (Conversion Optimizer) - use CPA bids
Advanced options
Bid type
Max CPA: Most you would want to pay for any particular conversion
Target CPA: Average amount you would like to pay for each conversion
Set an initial target CPA (1-per-click) bid
Start with recommended bid: $13.20
This is approximately equivalent to a maximum CPC bid of $1.86
Start with custom bid $ Target CPA
After you click 'Save' you will be able to set individual ad group bids.
Your current maximum CPC bids will be saved so you can return to manual bidding later.
Save
Cancel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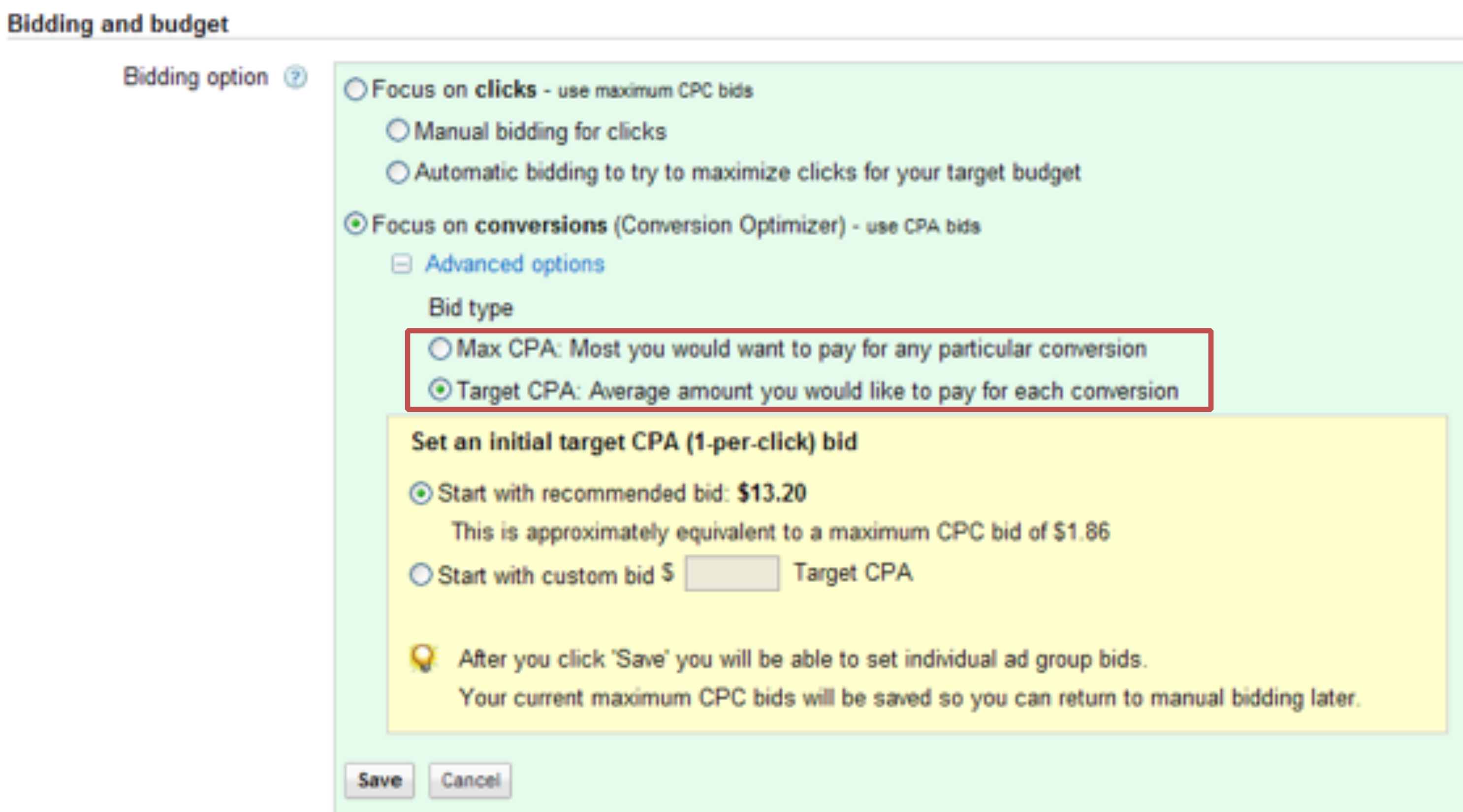
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- **Few techniques** for budgeted settings.
- [Ausubel], [Dobzinski, Lavi, Nisan]: clinching auctions
- Extended in many directions in previous years:
    - general environments: [Fiat et al], [Colini-Baldeschi et al], [Goel, Mirrokni, PL], [Dobzinski, PL]
    - revenue: [Bhattacharya et al], [Devanur, Ha, Hartline]
    - online settings: [Goel, Mirrokni, PL]

- **Two issues with current state of affairs:**
  - **Clinching** is all we know how to do
  - Our knowledge is (mostly) limited to **hard budget constraints**.

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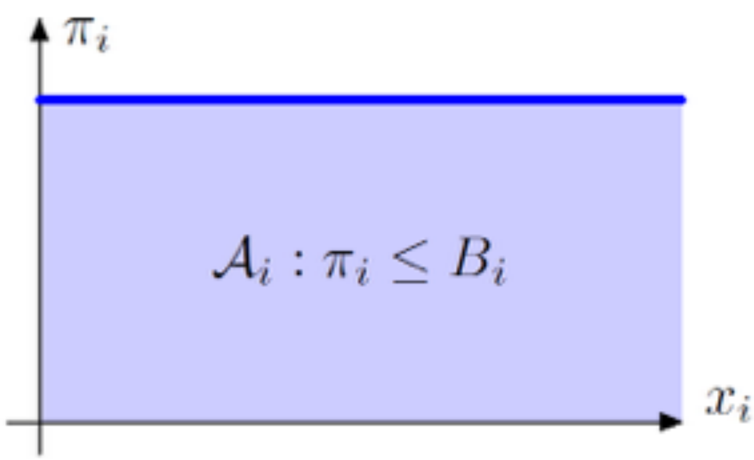
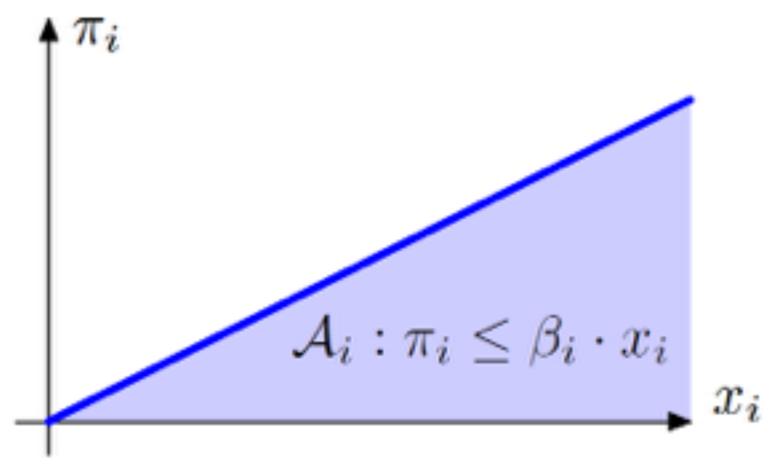
Plan: Address the second issue.

Hard Budgets: $u_i(x_i, \pi_i) = \begin{cases} v_i(x_i) - \pi_i, & \pi_i \leq B_i \\ -\infty, & \end{cases}$

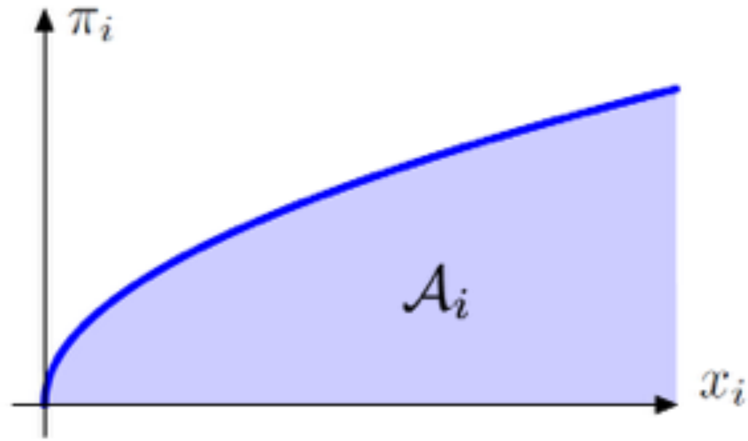
Average budgets: $u_i(x_i, \pi_i) = \begin{cases} v_i(x_i) - \pi_i, & \pi_i \leq \beta_i x_i \\ -\infty, & \text{o.w.} \end{cases}$

Generic constr: $u_i(x_i, \pi_i) = \begin{cases} v_i(x_i) - \pi_i, & \boxed{(x_i, \pi_i) \in \mathcal{A}_i} \\ -\infty, & \text{o.w.} \end{cases}$





Generic admissible set:



- $(0,0) \in \mathcal{A}_i$

- right-down closeness

$$(x_i, \pi) \in \mathcal{A}_i \Rightarrow (x_i + \delta, \pi_i - \delta') \in \mathcal{A}_i, \forall \delta, \delta' \geq 0$$

- convexity: distributions over admissible outcomes are admissible

- topological closeness

Setting

- n agents with (private) value v_i per item (say clicks) and (public) admissible set $\mathcal{A}_i$
- allocation constraints $\mathbf{P} \subseteq \mathbb{R}^n$ (polymatroid) i.e. sponsored search, one-sided-matching, flows, spanning trees, ...

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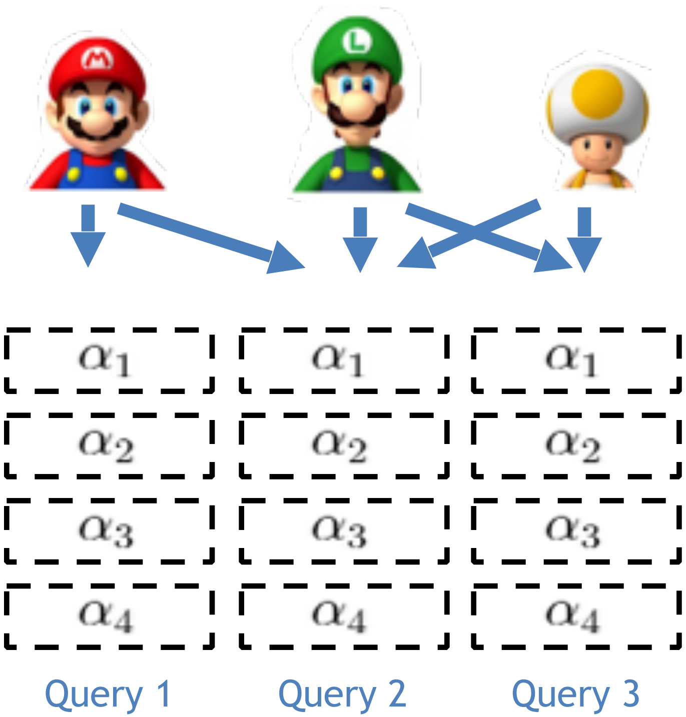

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Goal

- truthful auction
- admissible outcomes
- Pareto efficient: no alternative outcome where each agent and the auctioneer weakly improve and at least one strictly improves.

Technique : Ausubel's clinching framework, following [Dobzinski, Lavi, Nisan], [Goel, Mirrokni, PL, 2012]

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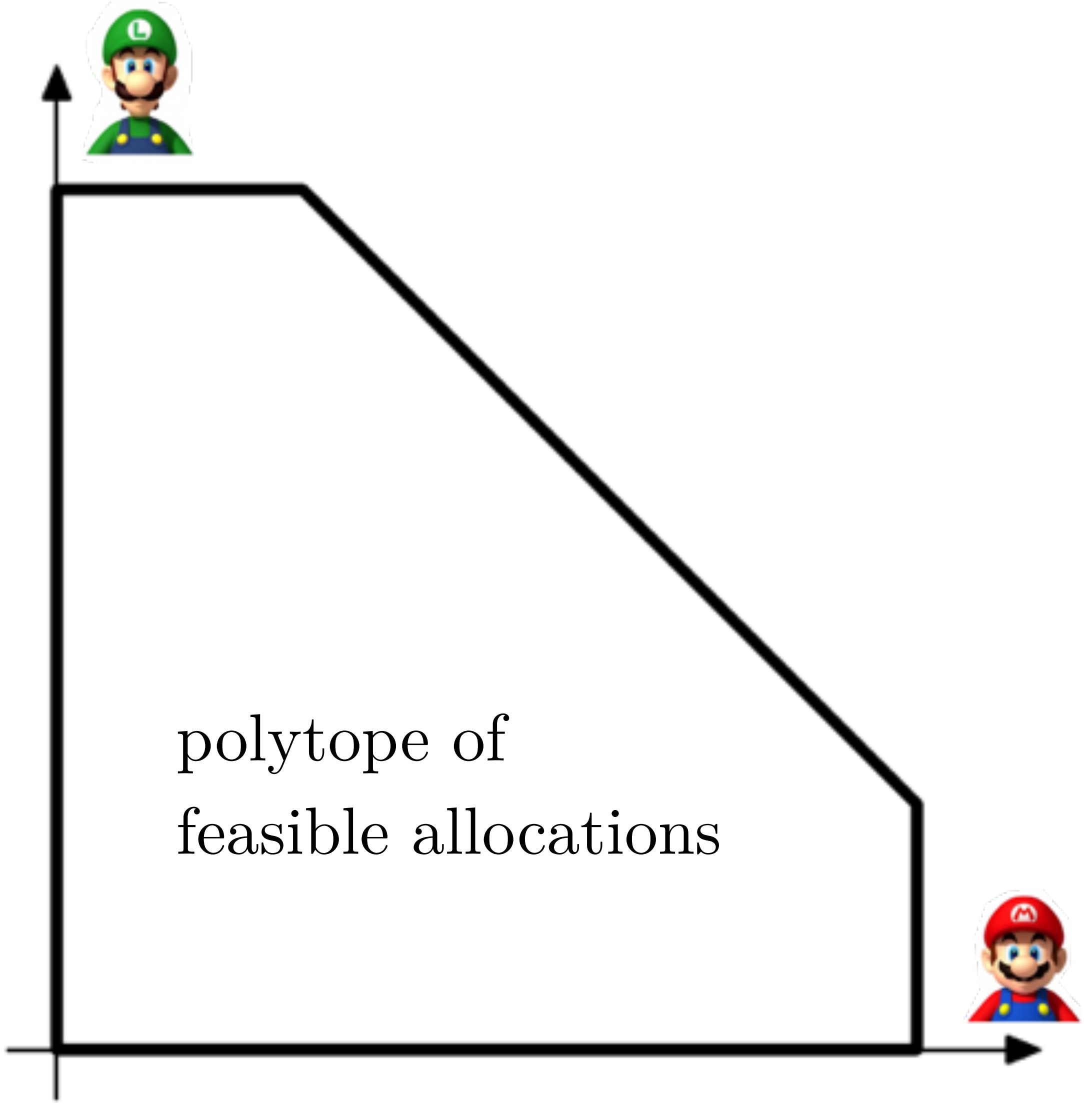

price
clock

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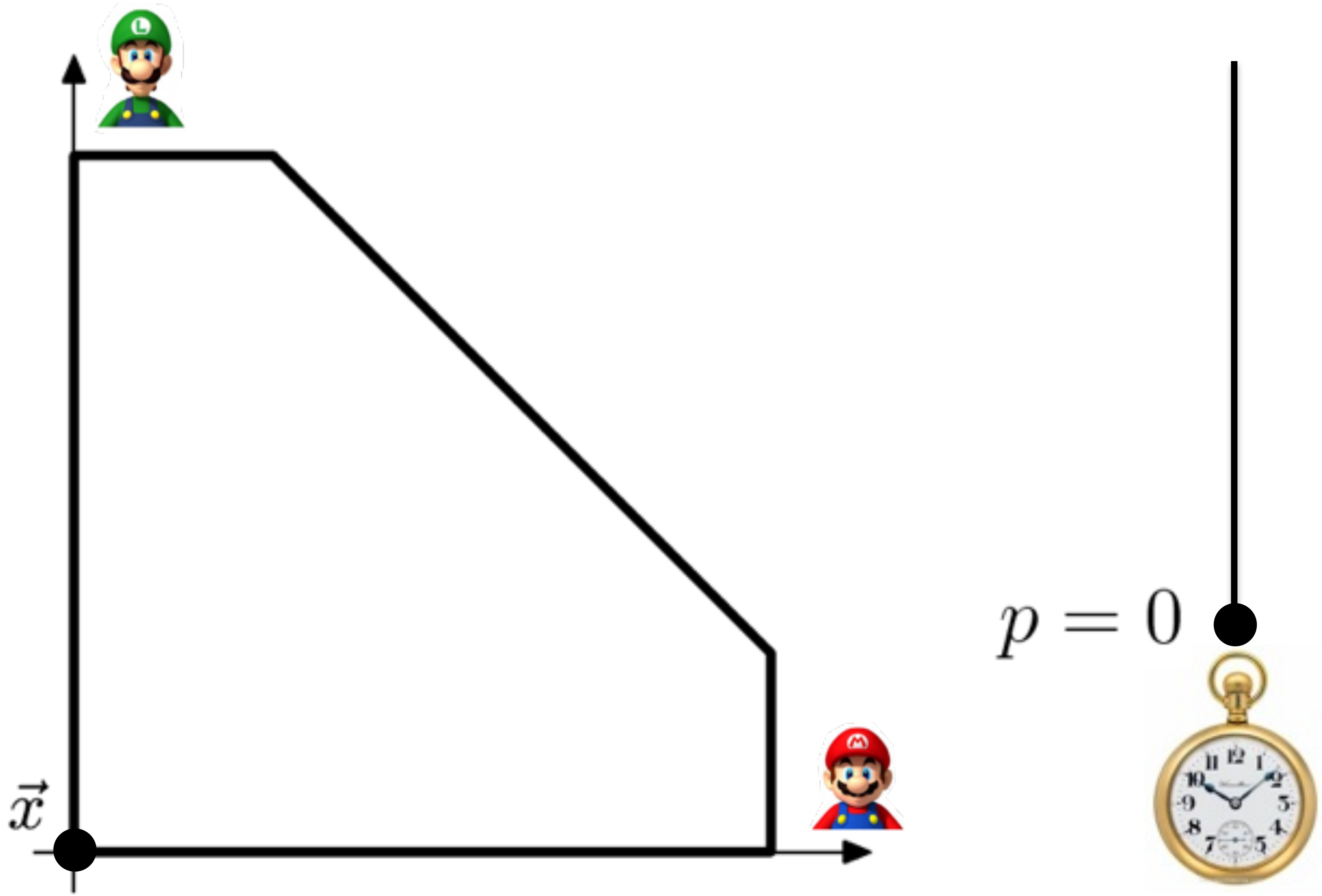

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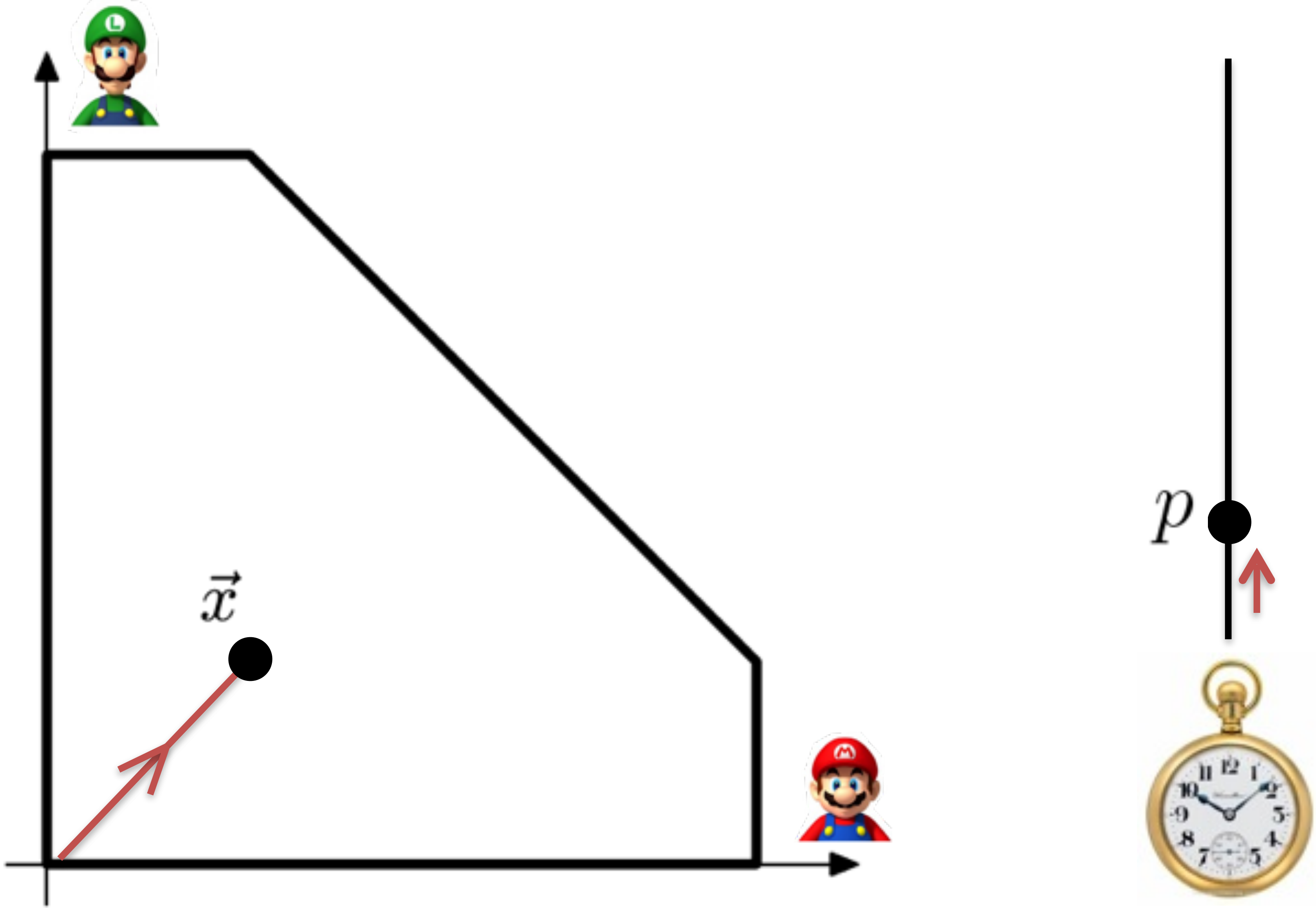

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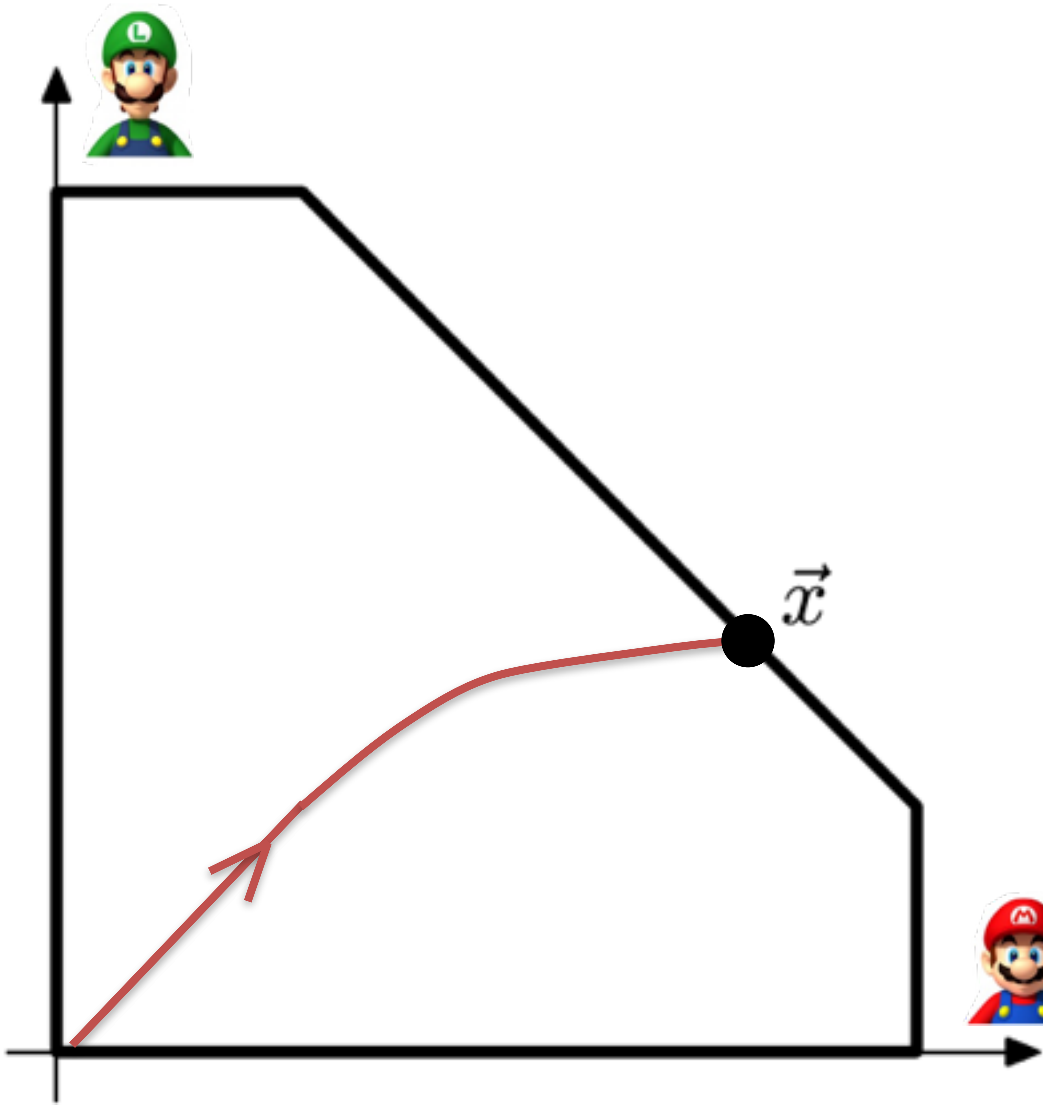

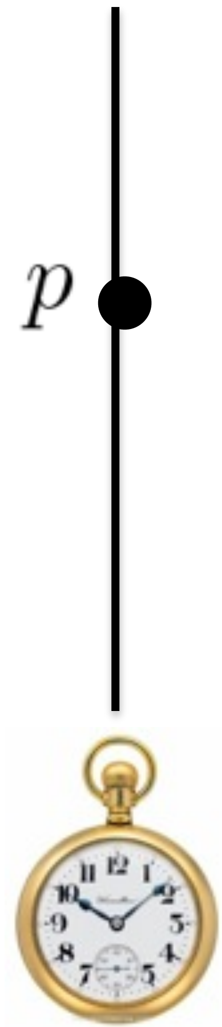

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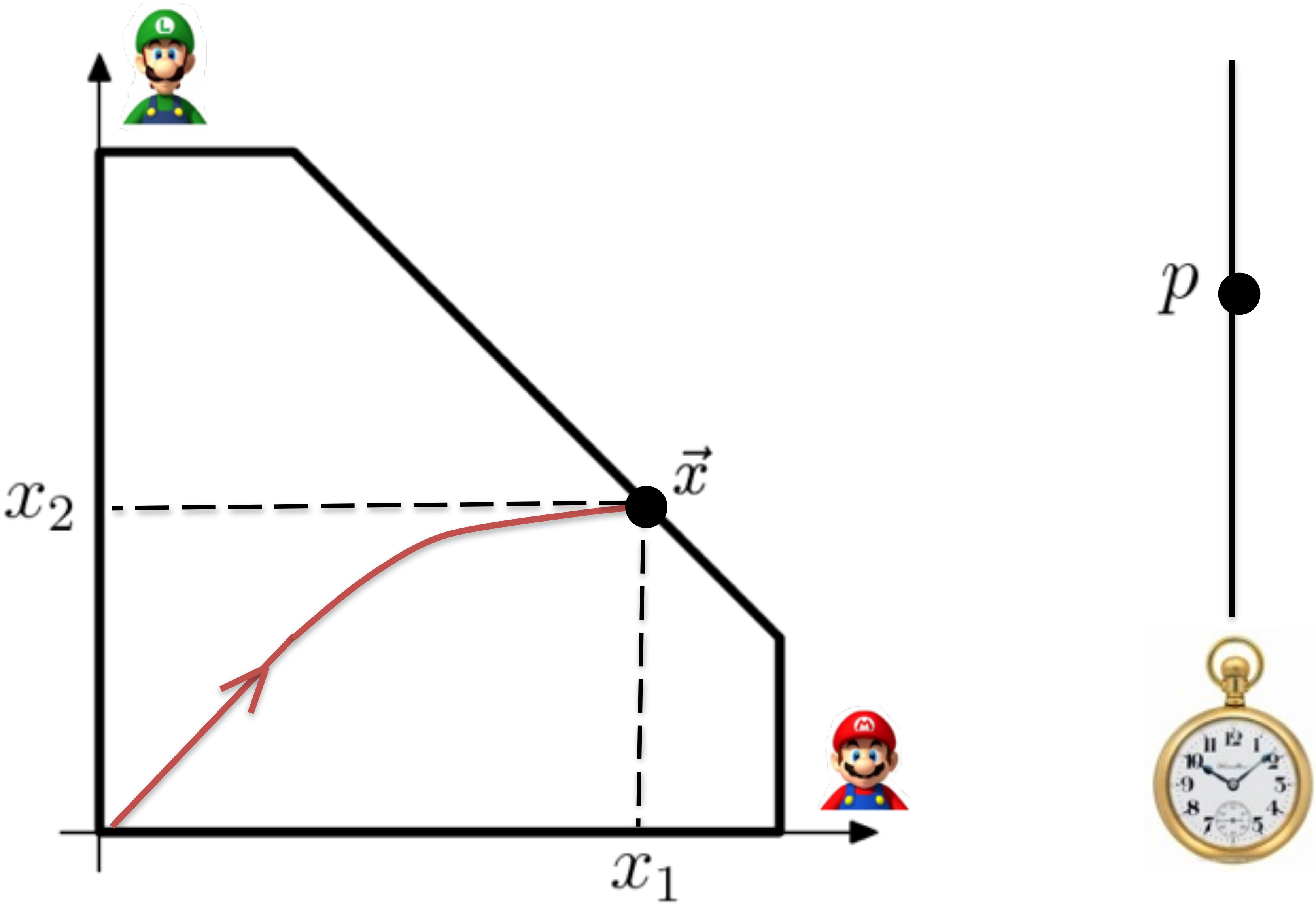

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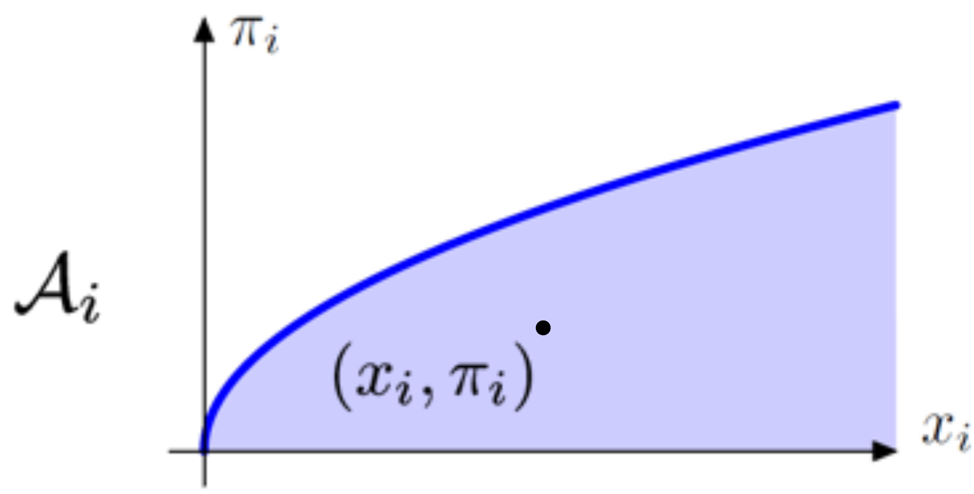
We initialize $(x_i = 0, \pi_i = 0)$ and set prices and update for all prices p

For each price p we compute the demands of each agent



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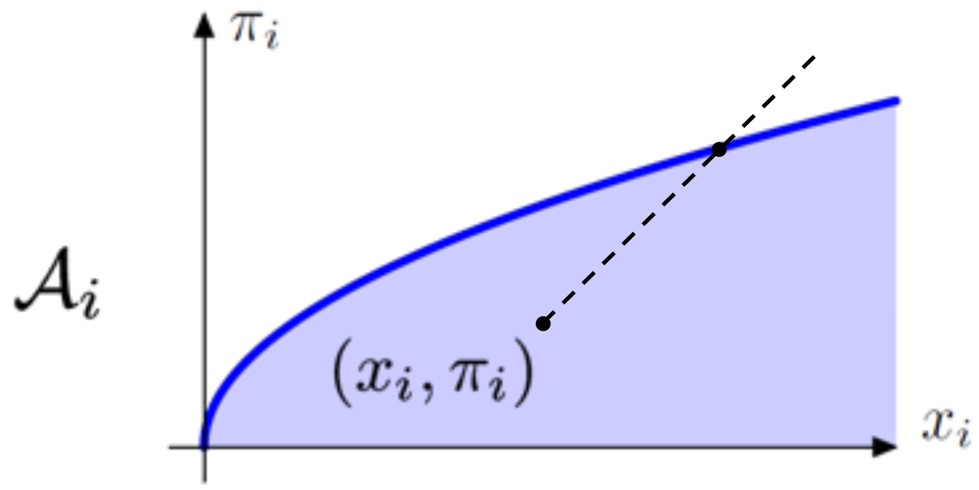

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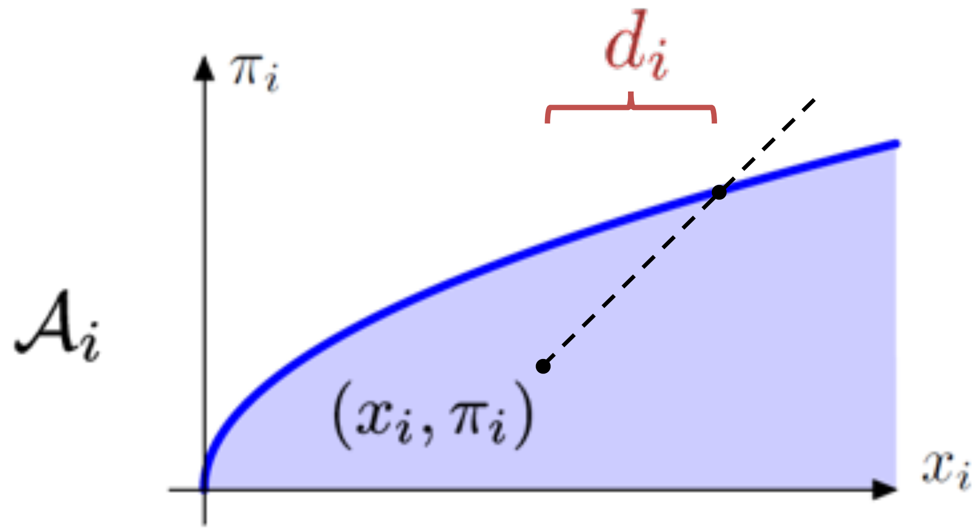

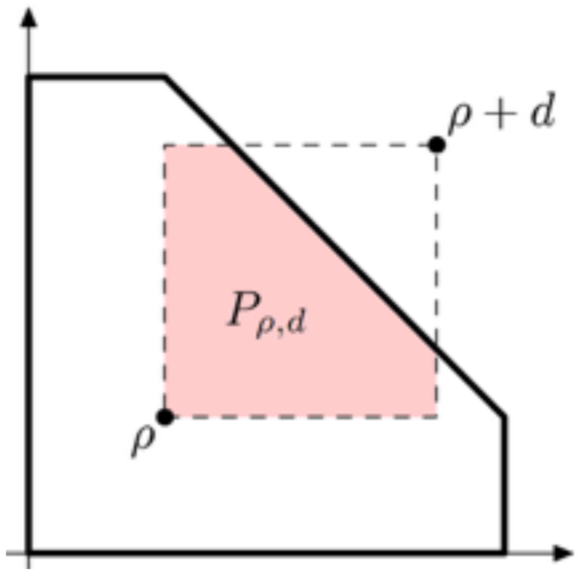

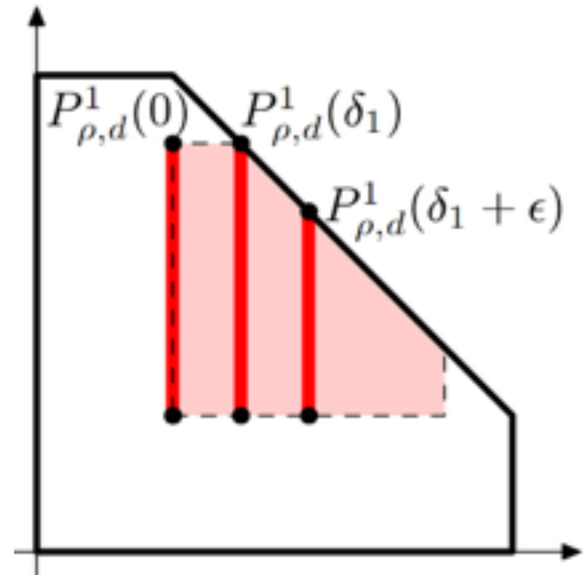

Clinching: find for each agent maximum amount δ_i that one can allocate to him without making the allocations of the other players infeasible.

$$x_i = x_i + \delta_i \qquad \pi_i = \pi_i + p\delta_i$$

Thm: The polyhedral clinching auction is truthful, admissible and Pareto-optimal.

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various new techniques needed to prove Pareto-optimality for generic $\mathcal{A}_i$:

Pareto optimality $\equiv$ no trade

$v_i \leq p \leq v_j$

$v_i < v_j$ $\quad$ $i \rightarrow j$

$x_i = x_i - \delta$ $\quad$ $x_j = x_j + \delta$

$\pi_i = \pi_i - p\delta$ $\quad$ $\pi_j = \pi_j + p\delta$

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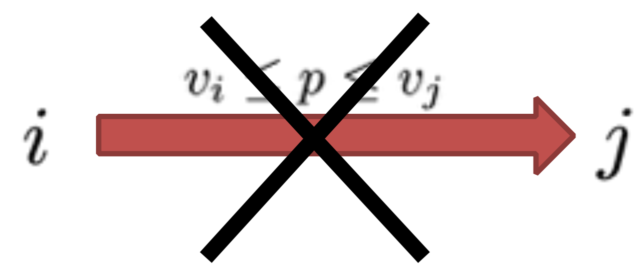
1) New outcome not admissible for j

2) Violates feasibility constraints



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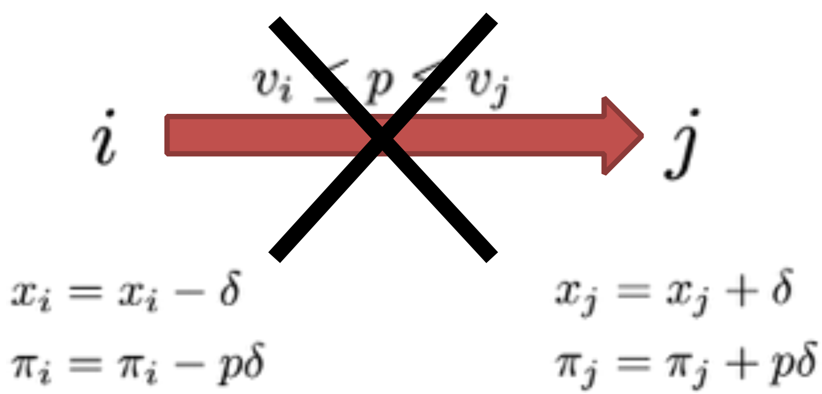
Hard budgets: no trade at one price p means not trade at any price



not true anymore...

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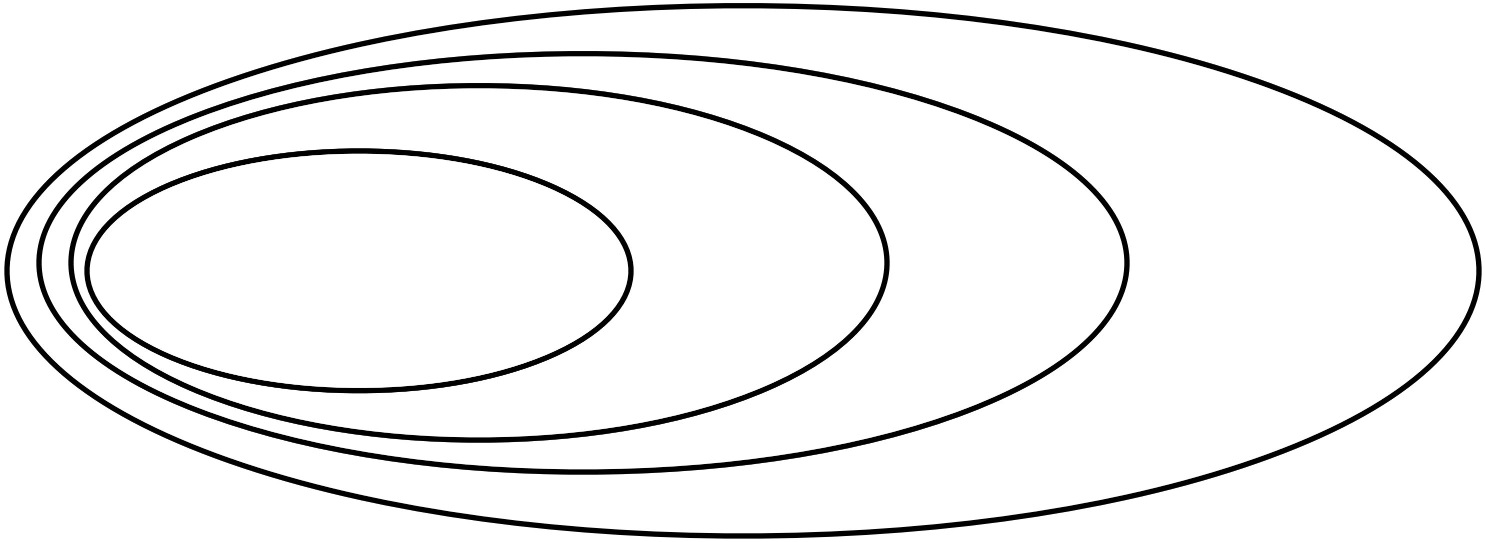
Structure of tight sets lemma: sets of agents



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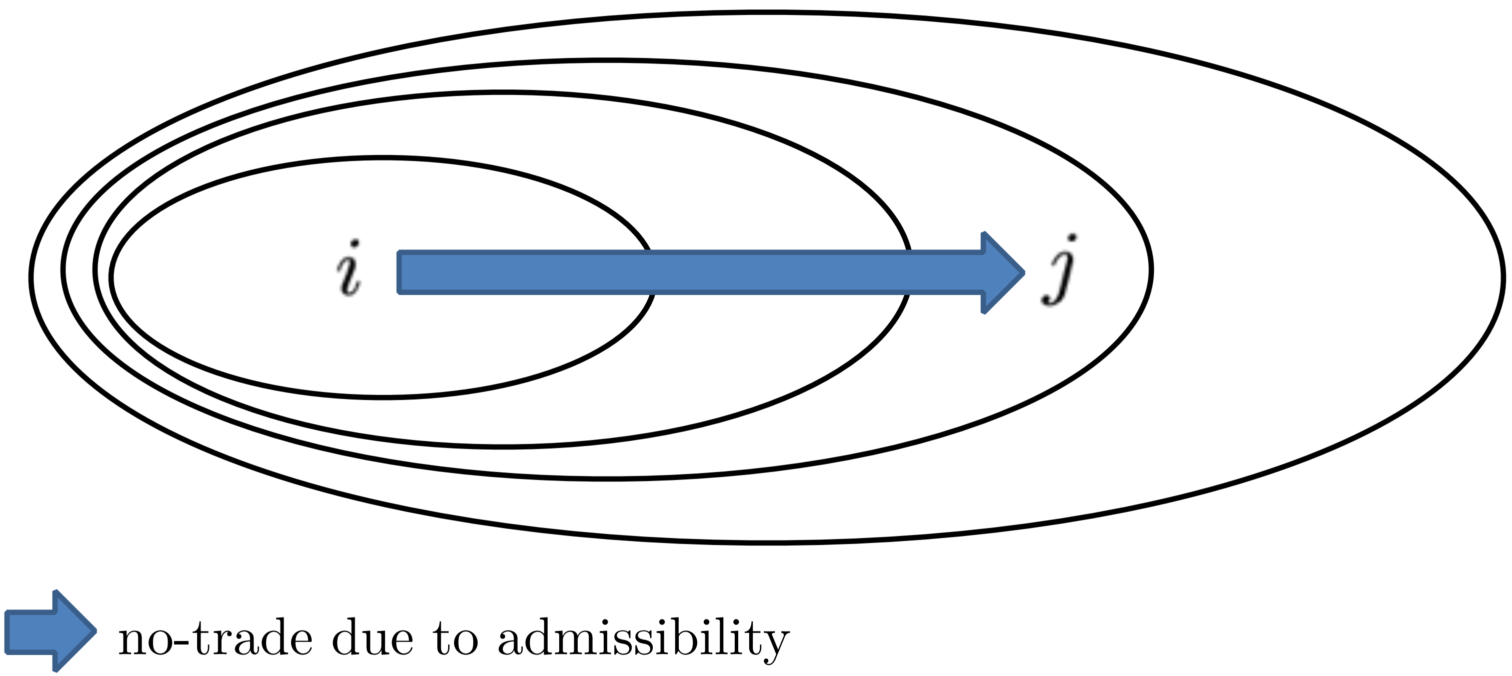

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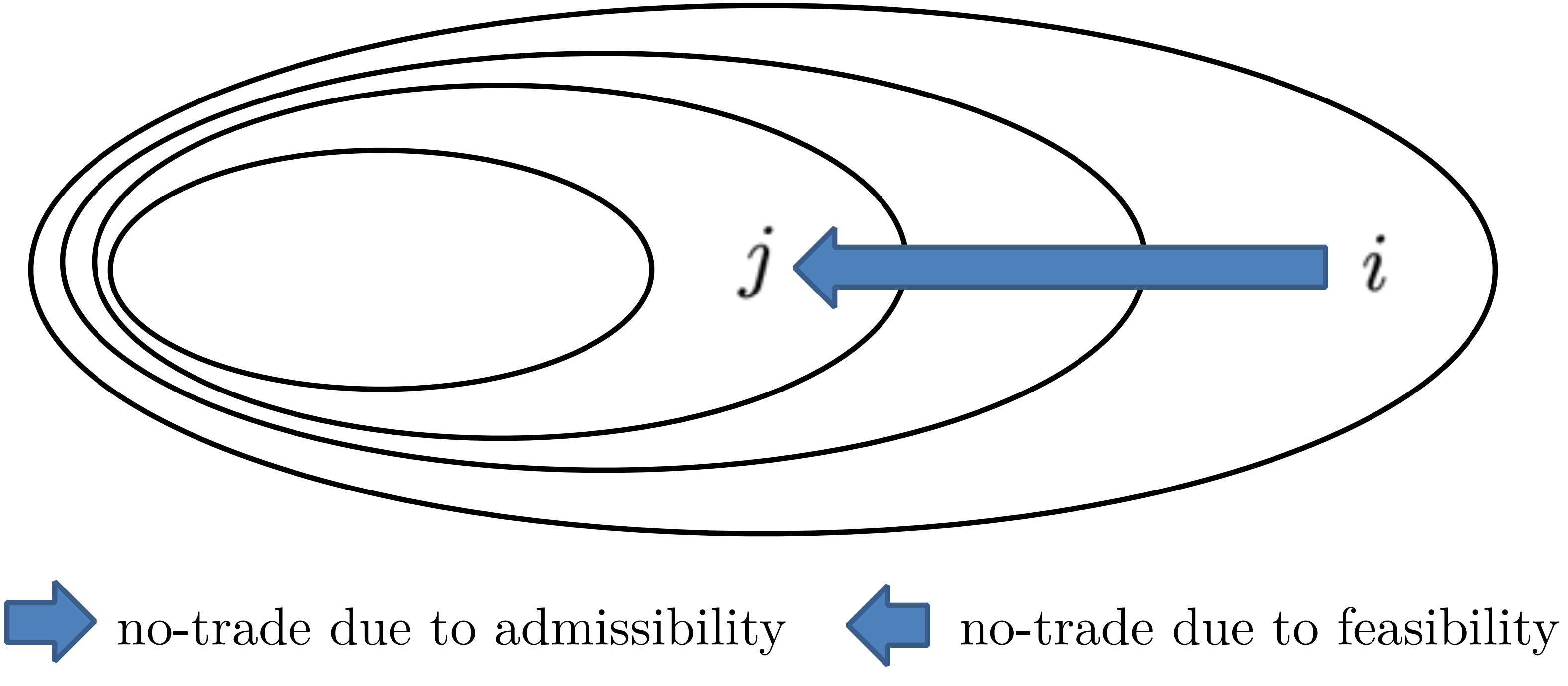

Future directions

How much further can clinching take us in non-quasilinear settings ?

Average budgets in online settings.

Heuristics in practice inspired by this auction.

Can we go **beyond clinching** ?

Thanks !