Which information to reveal in the interface of AdExchange and how does that affect revenue and welfare?
web surfers = { }
web surfers = \{ p_1, p_2, p_3, p_4, p_5 \}
Parties Head to Showdown as Obama Warns of a ‘Crisis’

By CARL HULSE and JACKIE CALMES 30 minutes ago

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- DealBook: Debt Drama Blocks Out Big Picture on Credit
- Gotham: Government Can’t Help? Tell That to the South Bronx
- Video: President Obama’s Address (MSNBC.com)
- Post a Comment | Read (801)

NEWS ANALYSIS
A ‘Unique Opportunity’ on the Debt Ceiling, Lost
By JACKIE CALMES & 53 PM ET

Whatever deal Congress and President Obama devise, it will probably fall short of the one nearly struck last week.

- FiveThirtyEight: Is Wall Street Out of Touch? 11:44 PM ET

OPINION
THE DEBT STALEMATE
Editorial: Republican Wreckage
Their latest actions push the nation to the brink of default.

Brook: Congress Leads
As the Old Guard takes the lead, President Obama has faltered.

- Nocera: Wells Fargo’s Pass
- Cohen: Breivik’s Enablers
- Mike Mullen: Trust With China
- Op-Ed: Terror Within

MARKETS
At 12:18 AM ET

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MARKETS

10:18 AM ET

JAPAN     CHINA

Nihkei  HangSeng  Shanghai

16,123.26  22,582.65  2,690.33
+73.15  +209.36  +1.58
+0.43%  +0.94%  +0.06%

Data delayed at least 15 minutes

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ad slot

AdExchange holds a second price auction
AdExchange holds a second price auction.
AdExchange holds a second price auction

Their value depends who is the user behind the impression.
web surfers = \{ p_1, p_2, p_3, p_4, p_5 \}

Tiffany & Co.

5  0.1  15  10  20
\[ \text{web surfers} = \{ p_1, p_2, p_3, p_4, p_5 \} \]

<table>
<thead>
<tr>
<th>Tiffany &amp; Co.</th>
<th>5</th>
<th>0.1</th>
<th>15</th>
<th>10</th>
<th>20</th>
</tr>
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<tbody>
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<td>Pop Art Supplies</td>
<td>25</td>
<td>10</td>
<td>0.1</td>
<td>0.1</td>
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</tr>
</tbody>
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web surfers = \{ P_1, P_2, P_3, P_4, P_5 \}

<table>
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<tr>
<th>Store</th>
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<tr>
<td>Music Store</td>
<td>10</td>
<td>20</td>
<td>1</td>
<td>5</td>
<td>0.2</td>
</tr>
</tbody>
</table>
web surfers = \{ P_1, P_2, P_3, P_4, P_5 \}

Tiffany & Co.

Pop Art Supplies

Music Store
Who knows what?

• AdExchange knows who is the user $j$ issuing the click

• Advertisers just know the prior $p$
One idea: revealing all the information

- Advertiser $i$ bids $v_i(j)$
- Revenue = $\sum_j p(j) \max 2_i v_i(j)$
One idea: revealing all the information

- Advertiser i bids $v_i(j)$

- Revenue = $\sum_j p(j) \max 2_i v_i(j)$

- Many problems:
  - Cherry picking
  - Revenue collapse
  - Adverse selection
  - Too much cognitive burden
\[
\text{web surfers} = \begin{bmatrix}
\text{Pop Art Supplies} & 25 & 0.1 & 0.1 & 0.1 & 0.1 \\
\text{Music Store} & 0.1 & 25 & 1 & 5 & 0.2 \\
\text{Tiffany & Co.} & 0.1 & 0.1 & 15 & 15 & 15 \\
\end{bmatrix}
\]
web surfers = \{ p_1 + p_2, p_3, p_4, p_5 \}

<table>
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<tr>
<th>Store</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
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<td>Pop Art Supplies</td>
<td>13</td>
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Other idea: bundling the items

- Group the items in sets $S_1 \ldots S_n$

- Revenue $= \sum_t \max_{2i} \sum_{j \in S_t} p(j)v_i(j)$
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- Revenue $= \sum_t \max_{2i} \sum_{j \in S_t} p(j)v_i(j)$

- [Ghosh, Nazerzadeh, Sundarajan ‘07]
- [Emek, Feldman, Gamzu, Tennenholtz ‘11]
  - strongly NP-hard to optimize revenue
  - 2-approximation
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Integral Partitioning Problem
Bundling the items fractionally
Bundling the items fractionally

==

Signaling
Bundling the items fractionally

=

Signaling

• [Emek, Feldman, Gamzu, Paes Leme, Tennenholtz ’12]
• [Bro Miltersen, Sheffet ‘12]
Signaling

• Design a signal $\sigma$ which is a random variable correlated with $j$
Signaling

- Design a signal \( \sigma \) which is a random variable correlated with \( j \)

- \( \sigma \in [s] \) and is represented by a joint probability \( \mathbb{P}(j, \sigma) \)
Signaling

- Design a signal $\sigma$ which is a random variable correlated with $j$

- $\sigma \in [s]$ and is represented by a joint probability $\mathbb{P}(j, \sigma)$

$$\begin{bmatrix}
\vdots & \vdots & \vdots \\
\vdots & \mathbb{P}(j, \sigma) & \vdots \\
\vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots \\
\end{bmatrix}$$

$$\sum_\sigma \mathbb{P}(j, \sigma) = p(j)$$
Signaling

• For user $j$, the search engine samples $\sigma$ according to

$$P(\sigma | j) = \frac{P(j, \sigma)}{\sum_{j'} P(j', \sigma)}$$

• Advertiser use $\sigma$ to update their bid

$$b_i = \mathbb{E}[v_i(j) | \sigma] = \frac{\sum_j v_i(j) P(j, \sigma)}{\sum_j P(j, \sigma)}$$
j = 3
\[ j = 3 \]

\[
\begin{align*}
\mathbb{P}(\sigma_1 | j = 3) \\
\mathbb{P}(\sigma_2 | j = 3) \\
\mathbb{P}(\sigma_3 | j = 3) \\
\mathbb{P}(\sigma_4 | j = 3)
\end{align*}
\]
j = 3
j = 3

\[
p'_1 | \square
\]

\[
p'_2 | \square
\]

\[
p'_3 | \square
\]

\[
p'_4 | \square
\]

\[
p'_5 | \square
\]
Signaling

- Expected revenue: $\sum_{\sigma} P(\sigma) \max_{i} E[v_i(j)|\sigma]$
Signaling

- Expected revenue: \[ \sum_{\sigma} \mathbb{P}(\sigma) \max_{i} 2_i \mathbb{E}[v_i(j)|\sigma] \]
  \[= \sum_{\sigma} \max_{i} \sum_{j} v_i(j) \mathbb{P}(j, \sigma) \]
Signaling

- Expected revenue: \( \sum_{\sigma} \mathbb{P}(\sigma) \max_{2i} \mathbb{E}[v_i(j)|\sigma] \)
  \[= \sum_{\sigma} \max_{2i} \sum_j v_i(j) \mathbb{P}(j, \sigma) \]

- How big does s (size of signaling space) need to be?

- How to optimize revenue? (\( \max_2 \) is not convex)
Theorem: If there are $n$ advertisers, we just need to keep $n(n-1)$ signals. One correspond to each pair of advertisers $(i_1, i_2)$.
Signaling

- Theorem: If there are $n$ advertisers, we just need to keep $n(n-1)$ signals. One correspond to each pair of advertisers $(i_1, i_2)$

\[
\begin{align*}
\max \sum_{i_1, i_2 \in [n], i_1 \neq i_2} R(\sigma_{i_1, i_2}) \quad \text{s.t.} \\
R(\sigma_{i_1, i_2}) &\leq \sum_{j \in [m]} \mathbb{P}(j, \sigma_{i_1, i_2}) \cdot v_{i_1}(j) \quad \forall i_1, i_2 \in [n], i_1 \neq i_2 \\
R(\sigma_{i_1, i_2}) &= \sum_{j \in [m]} \mathbb{P}(j, \sigma_{i_1, i_2}) \cdot v_{i_2}(j) \quad \forall i_1, i_2 \in [n], i_1 \neq i_2 \\
\sum_{i_1, i_2 \in [n], i_1 \neq i_2} \mathbb{P}(j, \sigma_{i_1, i_2}) &= p(j) \quad \forall j \in [m] \\
\mathbb{P}(j, \sigma_{i_1, i_2}) &\geq 0 \quad \forall i_1, i_2 \in [n], i_1 \neq i_2, \forall j \in [m].
\end{align*}
\]
Signaling

- Theorem: The revenue-optimal signaling can be found in polynomial time.

- Also, there is an optimal signaling scheme that preserves $\frac{1}{2}$ of the optimal social welfare.
Signaling

- Theorem: The revenue-optimal signaling can be found in polynomial time.

- Also, there is an optimal signaling scheme that preserves $\frac{1}{2}$ of the optimal social welfare.

- It improves the optimal (integral) bundling up to a factor of 2.
Signaling in a Bayesian World

- Valuations of advertiser $i$ for user $j$ depends on some unknown state of the world $\omega \sim q$

$$v_i(j, \omega)$$
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\[ v_i(j, \omega) \]

• Let $k = \text{supp}(q)$
Signaling in a Bayesian World

- Valuations of advertiser $i$ for user $j$ depends on some unknown state of the world $\omega \sim q$
  $$v_i(j, \omega)$$

- Let $k = \text{supp}(q)$

- We can find the optimal signaling scheme in polynomial time if $k = O(1)$
  - Naïve extension of the full information LP
Signaling in a Bayesian World

• If $m$ (number of user types) is constant, then we can find the optimal signaling scheme in time polynomial in $k,n$.

• Geometry of hyperplane arrangements
Signaling in a Bayesian World

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• \textbf{NP-hard}: \( n=3 \) and arbitrary \( m, k \)
Signaling in a Bayesian World

- If \( m \) (number of user types) is constant, then we can find the optimal signaling scheme in time polynomial in \( k,n \).
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- **NP-hard**: \( n=3 \) and arbitrary \( m,k \)

- **Open**: approximability of this problem
Open Problems

Approximability in the Bayesian Case
Open Problems

Approximability in the Bayesian Case

Bayesian case with independent values
Open Problems

Approximability in the Bayesian Case

Bayesian case with independent values

Optimal auctions with signaling
Thanks!