POLYHEDRAL CLINCHING
AUCTIONS BEYOND HARD BUDGET CONSTRAINTS

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• Item values are an useful abstraction but often intangible.

• Typically, buyers care about the items (impressions) only in aggregate.

• Aggregate statistics about an auction result: budget spent, average cpc, ...
Welcome to AdWords!

Create your first campaign

Getting started

1. Choose your budget
2. Create your ads
3. Select keywords that match your ads to potential customers
4. Enter your billing information.
Bidding and budget

Bidding option
- [ ] Basic options
- [ ] Advanced options
- [ ] I'll manually set my bids for clicks
- [x] AdWords will set my bids to help maximize clicks within my target budget

Budget
- $ __________ per day
  Actual daily spend may vary.
Bidding and budget

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  - I’ll manually set my bids for clicks
  - AdWords will set my bids to help maximize clicks within my target budget

Budget
- $____ per day
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Bidding and budget

Bidding option
- Focus on clicks - use maximum CPC bids
- Manual bidding for clicks
- Automatic bidding to try to maximize clicks for your target budget

- Focus on conversions (Conversion Optimizer) - use CPA bids
  - Advanced options
    - Bid type
      - Max CPA: Most you would want to pay for any particular conversion
      - Target CPA: Average amount you would like to pay for each conversion

Set an initial target CPA (1-per-click) bid
- Start with recommended bid: $13.20
  This is approximately equivalent to a maximum CPC bid of $1.86
- Start with custom bid $____ Target CPA

After you click ‘Save’ you will be able to set individual ad group bids.
Your current maximum CPC bids will be saved so you can return to manual bidding later.

Save  Cancel
Bidding and budget

Bidding option
- Basic options
  - I’ll manually set my bids for clicks
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Advanced options

Bidding and budget

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- Focus on clicks - use maximum CPC bids
  - Manual bidding for clicks
  - Automatic bidding to try to maximize clicks for your target budget
- Focus on conversions (Conversion Optimizer) - use CPA bids
  - Advanced options
    - Max CPA: Most you would want to pay for any particular conversion
    - Target CPA: Average amount you would like to pay for each conversion

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Save | Cancel
• Few techniques for budgeted settings.

• [Ausubel], [Dobzinski, Lavi, Nisan]: clinching auctions

• Extended in many directions in previous years:
  
  • general environments: [Fiat et al], [Colini-Baldeschi et al], [Goel, Mirrokni, PL], [Dobzinski, PL]
  
  • revenue: [Bhattacharya et al], [Devanur, Ha, Hartline]

  • online settings: [Goel, Mirrokni, PL]
• Two issues with current state of affairs:
  • Clinching is all we know how to do
  • Our knowledge is (mostly) limited to hard budget constraints.
Two issues with current state of affairs:

• **Clinching** is all we know how to do

• Our knowledge is (mostly) limited to **hard** budget constraints.

Plan: Address the second issue.
Hard Budgets: \[ u_i(x_i, \pi_i) = \begin{cases} v_i(x_i) - \pi_i, & \pi_i \leq B_i \\ -\infty, & \text{otherwise} \end{cases} \]

Average budgets: \[ u_i(x_i, \pi_i) = \begin{cases} v_i(x_i) - \pi_i, & \pi_i \leq \beta_i x_i \\ -\infty, & \text{otherwise} \end{cases} \]

Generic constr: \[ u_i(x_i, \pi_i) = \begin{cases} v_i(x_i) - \pi_i, & (x_i, \pi_i) \in A_i \\ -\infty, & \text{otherwise} \end{cases} \]
Generic admissible set:

- $(0, 0) \in A_i$
- right-down closeness

$$(x_i, \pi) \in A_i \Rightarrow (x_i + \delta, \pi_i - \delta') \in A_i, \forall \delta, \delta' \geq 0$$
- convexity: distributions over admissible outcomes are admissible
- topological closeness
Setting

- $n$ agents with (private) value $v_i$ per item (say clicks) and (public) admissible set $A_i$
- allocation constraints $P \subseteq \mathbb{R}^n$ (polymatroid)
  i.e. sponsored search, one-sided-matching, flows, spanning trees, ...
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Goal

• truthful auction
• admissible outcomes
• Pareto efficient: no alternative outcome where each agent and the auctioneer weakly improve and at least one strictly improves.
Technique: Ausubel’s clinching framework, following [Dobzinski, Lavi, Nisan], [Goel, Mirrokni, PL, 2012]
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We initialize \( (x_i = 0, \pi_i = 0) \) and set prices and update for all prices \( p \)

For each price \( p \) we compute the demands of each agent
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For each price \(p\) we compute the demands of each agent.
Clinching: find for each agent maximum amount $\delta_i$ that one can allocate to him without making the allocations of the other players infeasible.

$$x_i = x_i + \delta_i \quad \pi_i = \pi_i + p\delta_i$$
Thm: The polyhedral clinching auction is truthful, admissible and Pareto-optimal.
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Various new techniques needed to prove Pareto-optimality for generic $A_i$:

Pareto optimality $\equiv$ no trade

\[
v_i < v_j \quad \Rightarrow \quad x_i = x_i - \delta \quad \pi_i = \pi_i - p\delta \quad x_j = x_j + \delta \quad \pi_j = \pi_j + p\delta
\]
**Thm:** The polyhedral clinching auction is truthful, admissible and Pareto-optimal.

**various new techniques needed to prove Pareto-optimality for generic $A_i$:**

Pareto optimality $\equiv$ no trade

1) New outcome not admissible for $j$

2) Violates feasibility constraints

\[ x_i = x_i - \delta \quad \pi_i = \pi_i - p\delta \]
\[ x_j = x_j + \delta \quad \pi_j = \pi_j + p\delta \]
**Thm:** The polyhedral clinching auction is truthful, admissible and *Pareto-optimal*.

Various new techniques needed to prove Pareto-optimality for generic $A_i$:

Pareto optimality $\equiv$ no trade

Hard budgets: no trade at one price $p$ means not trade at any price

\[
\begin{align*}
x_i &= x_i - \delta \\
\pi_i &= \pi_i - p\delta \\
x_j &= x_j + \delta \\
\pi_j &= \pi_j + p\delta
\end{align*}
\]

Not true anymore...
**Thm:** The polyhedral clinching auction is truthful, admissible and *Pareto-optimal.*

**Structure of tight sets lemma:** sets of agents
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Structure of tight sets lemma: sets of agents

no-trade due to admissibility
Thm: The polyhedral clinching auction is truthful, admissible and Pareto-optimal.

Structure of tight sets lemma: sets of agents

→ no-trade due to admissibility ← no-trade due to feasibility
Future directions

How much further can clinching take us in non-quasilinear settings?

Average budgets in online settings.

Heuristics in practice inspired by this auction.

Can we go beyond clinching?
Thanks !